SCIENCE

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THE INFLUENCE OF FOURIER'S SERIES UPON THE DEVELOPMENT OF MATHEMATICS[1]

IN selecting a subject for to-day's address I have had the difficult task of interesting two distinct classes of men, the astronomer and the mathematician. I have therefore chosen a topic which, I trust, will appeal to both—trigonometric series. Though I propose to treat it only in its mathematical aspects, I shall try to do so in a broad way, tracing its *general* influence upon the trend of mathematical thought.

As you know, the theory of the infinite trigonometric series,

$$\text{(I.)}\quad f(x) = \tfrac{1}{2}a_0 + (a_1 \cos x + b_1 \sin x) + (a_2 \cos 2x + b_2 \sin 2x) + \cdots$$

is different *ab initio* from that of the power series,

$$P(x) = c_0 + c_1(x-a) + c_2(x-a)^2 + \cdots.$$

For the latter the fundamental element is x^n, of which the graph is, for positive x, a monotone increasing function, wholly regular, without peculiarities of any sort. It is therefore in no way surprising that the power series obtained by combining terms of form $c_n x^n$ define the most civilized members of mathematical society—the so-called analytic functions—which are most orderly in their behavior, being continuous throughout their "domains," possessing derivatives of all orders and a Taylor's series at every point; and so forth. On the other hand, the graph of $\sin nx$ or $\cos nx$ is a wave curve with crests and troughs, whose number in any x interval increases indefi-

[1] Address of the vice-president of Section A—Mathematics and Astronomy, American Association for the Advancement of Science, Atlanta, 1913.

nitely with n. Accordingly, the functions defined by infinite trigonometric series are obtained by compounding waves of varying intensity and different wave-lengths and may be almost infinitely complicated in their behavior. This fact was fraught with vital consequences for mathematical development.

A further distinction between the trigonometric and power series appears in respect to the values which their argument may take. The convergent power series $P(x)$ has significance for at least a limited domain of imaginary values of x; on the other hand, it is possible for trigonometric series to define functions which have no meaning except for real values of x. As, therefore, the trigonometric series has a functional content totally different from that of the power series, its influence was felt first, and primarily, in the development of the notion of a function of a real variable.

The concept *function* was at first vague, as vague and indefinite as our geometrical intuitions. It had its root in the 17th century in the analytic geometry of Descartes. Here the variation of y with x along a curve inevitably suggests the notion of a function. The first published definition of the term appeared in 1718 when John Bernoulli defined a function of a variable as "*an expression which is formed in any manner from the variable and constants.*" Thirty years later, in his "Infinitesimal Analysis," Euler defined it in like manner except that the function is now an "*analytic expression.*" What is meant by "analytic expression" is not explained, but from his definition of special classes of functions it would appear that the term denoted an expression put together in terms of the variable and constants by a finite or infinite number of operations of addition, subtraction, mulitiplication, and division. Differentiation and integration were also undoubtedly permissible.

About this time there began the famous controversy over the mathematical representation of a vibrating string. This satisfies the well-known differential equation

$$\frac{\partial^2 w}{\partial t^2} = a^2 \frac{\partial^2 w}{\partial x^2},$$

where a is a certain constant, x the position of a particle on the string when taut, and w its transverse displacement at time t. A solution of this problem for the case of fixed end points was given by d'Alembert in 1747 under the form

$$w = f(x + at) - f(at - x),$$

where $f(x)$ denotes an arbitrary function whose nature he apprehended too narrowly. But he claimed to have the general solution inasmuch as his solution involved an *arbitrary* function.

This shot into mathematics the question: *What is an arbitrary function?* Even to-day this question is a vexing one, owing to disagreement in the point-set theory concerning certain principles of logic which cluster around the "*Princip der Auswahl*" as a center. But mathematicians had not then arrived at the subtleties of the present day. Their difficulties were really caused more by imperfect notions concerning a function than by the degree of arbitrariness. On the basis of the above definition of a function then current, Euler maintained that d'Alembert's solution was particular, rather than the most general possible. He rightly apprehended the nature of the physical problem and saw that the motion of the string subsequent to the initial instant was completely determined by the initial form of the string and the initial velocities of its points. Now the initial shape of the string could be a continuous geometrical curve composed of successive pieces whose forms are absolutely independent of one

another. To represent these pieces, Euler claimed that an equal number of different analytic expressions, or arbitrary functions, were necessary. Hence, as d'Alembert's solution involved only *one* arbitrary function, it could not be the general solution of the problem.

In these considerations of Euler there is a sharp antithesis between geometry and analysis. In Euler's thought the independent pieces of the above curve formed "*curvæ discontinuæ seu mixtæ seu irregulares.*" There was a blind belief that the definition of a curve in any interval by a mathematical expression carried with it a definite continuation of the curve beyond the interval, the violation of which was a violation of analysis. Thus the question was raised as to the relative power of mathematically constructible expressions and of geometric representation, and it was decided that geometric form transcends analytic expression rather than the converse.

The dual character of this controversy was changed into a triple one by Daniel Bernoulli, who first introduced Fourier's series into physics and obtained the solution of the equation of the vibrating string with fixed end points under the form of a trigonometric series,

$$y=\sum_{n=1}^{\infty} a_n \sin\frac{n\pi x}{l}\cos\frac{n\pi at}{l},$$

where l denotes the length of the string. The separate terms of this series give the tones and overtones of the vibrating string. Inasmuch as this solution is compounded of an infinite number of tones and overtones of all possible intensities, Daniel Bernoulli claimed that he had obtained the general solution of the problem.

For $t=0$ the above equation gives as the initial form of the string,

$$y=\sum_{n=1}^{\infty} a_n \sin\frac{n\pi x}{l}.$$

The question then at once arose whether d'Alembert's arbitrary function was capable of expansion into such a sine series. To Euler this seemed unthinkable. It was, so to speak, against the laws of the game, it was contrary to the rules of analysis that arbitrary, non-periodic functions could be represented in terms of periodic functions. Hence to Euler, Bernoulli's solution of the problem appeared even more limited than that of d'Alembert.

I have not the time to follow further this controversy, nor to show how d'Alembert and Lagrange united with Euler in declaring Daniel Bernoulli wrong in his claim. Yet not withstanding this overwhelming preponderance of authority Daniel Bernoulli was right. The controversy gradually languished without any clear conclusion till 1807, twenty-five years after Bernoulli's death, when Fourier presented to the French Academy one of the first of his communications which were summed up in 1822 in his "Analytic Theory of Heat." In this communication he startled Lagrange with the absolutely revolutionary doctrine that an arbitrarily given curve or function, irrespective of its nature, could be represented in any interval by a trigonometric series. Fourier sought no strict proof of his assertion, but the concrete examples which he gave vindicated its force. The precise limitations necessary to make the assertion exactly true remained, and to some extent still remain, for his successors to ascertain.

Fourier's result not merely vindicated Daniel Bernoulli's claim for his series, but showed that his claim fell far short of the reality. At a single blow it shattered hopelessly the notion of Euler and his contemporaries that a mathematical function could be carried continuously beyond the interval of definition in only a single way. But Fourier's examples went further than this. The arbitrary curve which he represented

by his series (I) could consist of separate pieces of any sort, not merely having no logical or definitional dependence on one another, but even not connecting successively at their ends. Thus by virtue of Fourier's assertion the power of representation through analytic expression is at least as great as the power of geometric picturization.

When once it was realized that mathematical expression could be adapted to the most diverse and unrelated demands upon it, no logical stopping-point could be seen short of the definition to-day accepted for a function of a real variable, and often referred to as the Dirichlet definition of a function. If, namely, to every value of x in an interval there corresponds a definite value of y (no matter how fixed or determined), y is called a function of x. For example, y may be equal to $+1$ at all rational points which are everywhere dense in any interval, and equal to 0 at the irrational points which are likewise everywhere dense. The Fourier series has thus necessitated a radical reconstruction of the notion of a function. *This is the first of its services which I wish to emphasize, the development and complete clarification of the concept of a function.*

Without loss of generality the interval in which the representation of the function by the series is required may be supposed to lie between $-\pi$ and $+\pi$. The series has then the form (I.) hitherto assumed. To determine its coefficients from the function Fourier used for the most part the equations,

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx dx,$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx dx;$$

but this determination, as Fourier himself stated, had been made by Euler before him. Trigonometric series whose coefficients can be obtained from the function represented in this manner are now called *Fourier's series* in distinction from trigonometric series whose coefficients can not be so obtained through integration. I have, however, in the title of my paper used the term "Fourier's series" in the older and broader sense as synonymous with all series of the form (I.).

The consideration of trigonometric series from a strict mathematical standpoint marks a second epoch in their history. This began with Dirichlet in 1829 in a memoir remarkable for its combination of clearness and rigor. Here he first determined accurately a set of sufficient conditions for the expansion of a function into a Fourier series. These familiar "Dirichlet conditions" it is scarcely necessary to repeat.

The extension of his results was at once sought, in particular by Riemann in a Göttingen Habilitations-Dissertation, which bore the title "Ueber die Darstellbarkeit einer Function durch eine trigonometrische Reihe." Riemann's aim was, however, to determine the *necessary* conditions for the representability of the function by the series. Must the function be integrable, as required in the sufficient conditions of Dirichlet? Must it have only a finite number of maxima and minima and of discontinuities? Such questions as these were easily answered by him in the negative, and a flood of light was poured upon the problem of representability but without making visible its complete solution. Possibly it was for this reason that this Habilitationsschrift, though delivered in 1854, was not published until thirteen years later, and then only after Riemann's death. Yet the work is a classic. As has been said of the poet Coleridge, so it could be said of Riemann, he wrote but little, but that little should be bound in gold.

To put the theory of Fourier's series on

a broader basis, Riemann perceived that first of all it was necessary to sharpen and widen the concept of an integral. Initially Leibnitz had thought of integration as a summation process, but this notion was forced into the background by its definition as the reverse of differentiation, until revived by Cauchy in 1823. He then defined the integral of a *continuous* function as most of us were taught to define it. The interval of integration was divided into n parts δ_i, each δ_i was multiplied by the value of the function $f(x_i)$ at its beginning, and the integral was defined as the limit of the sum $\Sigma\delta_i f(x_i)$ when the number of parts increased indefinitely, their size diminishing indefinitely. Because of the continuity of the function this definition of the definite integral was equivalent to that framed by means of the reverse process of differentiation. Riemann dismisses altogether the requirement of continuity for the function, and in forming the sum multiplies each subinterval δ_i by the value of the function, not necessarily at the beginning of the interval, but at a point ξ_i *arbitrarily* assumed in the subinterval. If, then, a limit exists for the sum $\Sigma\delta_i f(\xi_i)$, irrespective of the manner of partitioning the interval and of the choice of the points ξ_i, this is called the integral. Thus he redefined the fundamental concept of the integral calculus, making it entirely independent of the differential calculus. This definition, often referred to as the Riemann definition of an integral, has now become the universally accepted one and is the basis of scientific treatment of the integral calculus. *Thus a second service of Fourier's series has been in laying the foundation of the modern integral calculus, and in such wise that it bid fair to completely eclipse the differential calculus in importance and reach.*

Riemann's memoir may also be characterized as the beginning of a theory of the mathematically discontinuous. The work of Fourier had disclosed that mathematical expressions could portray functions with breaks, and the exacter but more limited investigation of Dirichlet drew still further attention to discontinuities. Riemann's definition of an integral did more; with one leap it planted the discontinuous function firmly upon the mathematical arena. In his integrable functions was comprised a class of functions whose discontinuities were infinitely dense in every interval, no matter how small—though indeed, as we now know, they are not totally discontinuous. One example which he gave was the integrable function defined by the convergent series,

$$1+\frac{(x)}{1^2}+\frac{(2x)}{2^2}+\frac{(3x)}{3^2}+\cdots,$$

in which (nx) denotes the positive or negative difference between nx and the nearest integral value, unless nx falls half way between two consecutive integers, when the value of (nx) is to be set equal to 0. The sum of the series was shown to be discontinuous for every rational value of x of the form $p/2n$, where p is an odd integer relatively prime to n.

This example and others, such as that of an integrable function with an infinite number of maxima and minima which was incapable of representation by a Fourier's series, were exceedingly stimulating. The investigation so impressed the imagination of Hermann Hankel as to call forth his notable memoir "Uber die unendlich oft oszillierenden und unstetigen Functionen" in which he unfolds his principle of "condensation of singularities," a memoir so important that it has even been said to "entitle him to be called the founder of the independent theory of functions of a real variable." It would appear to me that this distinction could be assigned with equal propriety to Riemann, for historically the

first of the two or three principal sources of this theory is to be found in Riemann's application of integration to discontinuous functions in his memoir on "the representability of a function by Fourier's series."

The above example of Riemann is notable for giving a mathematical expression for a discontinuous function incapable of graphical representation. I have already pointed out how Euler conceived of graphs so arbitrary as to be impossible of representation through an "analytic expression." The scales were now turned decisively to the other side, though it was not till later that it was recognized that our geometric figures have only an approximating character which our mathematical equations refine.

But the full power of mathematical expression was not realized until 1872–1875, when Weierstrass startled the mathematical world with an example (first published by Du Bois Reymond in 1875) of a continuous function having nowhere a derivative, or, in other terms, of a continuous curve without a tangent. The function given by Weierstrass was a trigonometric series

$$\sum_{n=1}^{\infty} b^n \cos a^n \pi x,$$

in which b is a positive constant less than 1 and a a fixed odd integer large enough to make ab exceed a certain value. Weierstrass states also that Riemann is supposed to have shown that the series

$$\Sigma \frac{\sin n^2 x}{n^2}$$

represented a function of like property, but the proof was not known. The failure of the continuous function of Weierstrass to be differentiable is due to the possession of an infinite number of maxima and minima in any interval, however small.

This example completed the separation between differentiable and continuous functions. It shows that *the former are only a subclass of the latter,* a result not even surmised by the boldest geometrical intuition. *This is the third influence of Fourier's series which I wish to emphasize.* So far as I know, this is the only one of its results which vitally affects geometric theory. It reveals the transcendence of analysis over geometrical perception. It signalizes the flight of human intellect beyond the bounds of the senses.

I return now to trace further the march of the function theory of the real variable. The second principal element in its formation seems to me to have been the concept of *uniform convergence.* This also seems to have been suggested chiefly by study of trigonometric series. Originally it was supposed that the sum of a convergent series of continuous functions shared the common properties of its terms and accordingly was continuous. Even so great a mathematician as Cauchy fell for a time into this error. The fallaciousness of this assumption was first pointed out by Abel in 1826 in his well-known memoir on the binomial series. Here he also discusses the series

$$f(\phi) = \sin \phi - \frac{\sin 2\phi}{2} + \frac{\sin 3\phi}{3} - \cdots, \qquad (2)$$

every term of which is continuous. Clearly the sum vanishes whenever ϕ is a multiple of π. If ϕ lies between $m\pi$ and $(m+1)\pi$, the sum is $\phi/2 - \nu\pi$, where ν denotes the

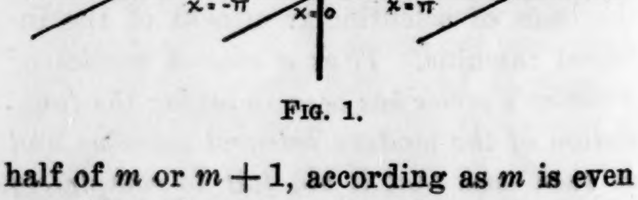

Fig. 1.

half of m or $m+1$, according as m is even or odd. Consequently, when ϕ passes through an odd multiple of π, the sum has a discontinuity of amount π, as is indicated in the adjoining graph. This re-

sult is in sharp contrast with the continuity of the sum which he demonstrates for the binomial and other real power series. At the same time he establishes the circular form of the region of convergence of the binomial series. Here, then, appears the initial cleavage between the theories of real and of analytic functions.

The difference between the trigonometric and the power series in respect to continuity is naturally to be sought in the character of the convergence at the points of continuity and of discontinuity. This difference was pointed out by Stokes in 1847 and by Seidel a year later. Both discovered the infinitely increasing slowness of convergence of the series on approaching a discontinuity of its sum. Consequently a discontinuity can not be enclosed in any interval, however small, in which the convergence is throughout "von gleichem Grade." In more modern parlance, the convergence is non-uniform. Seidel in his introduction explicitly points out that the erroneousness of Cauchy's conclusion (see above) is obvious from the existence of discontinuities in functions represented by Fourier's series, and he is evidently incited thereby to seek a cause for the discontinuity. The origin of Stokes's study is sufficiently obvious from its title: "On the Critical Values of the Sum of Periodic Series." His failure to appreciate the importance of his own convergence discussion is evident from the fact that it is not even mentioned in the opening analysis of his lengthy memoir.

A third discoverer of uniform convergence was Weierstrass, who is known to have been in possession of the notion as early as 1841. Through his followers (Heine and others) it gradually percolated into the mathematical literature. Unlike Seidel and Stokes, he thoroughly realized its importance. As Osgood has well said in his *Functionentheorie,* he developed uniform convergence into one of the most important organs "(methods) of modern analysis." The origin of the notion in the case of Weierstrass I have been unable to ascertain. A conjecture or surmise may therefore be pardoned. As is well known, the work of Weierstrass is rooted in that of Abel, the central theme or core being the theory of Abelian functions. It would not seem to be altogether improbable that both Weierstrass's theory of the analytic function and his concept of uniform convergence had as their starting point Abel's memoir on the binomial series. For here, on the one hand, with the demonstration of the circular form of the region of convergence of the binomial series, we find a proof of the continuity of the series which involves implicitly the idea of uniform convergence; on the other hand, we have in the footnote a series with discontinuities due, in fact, to non-uniform convergence. It would be a small matter for the discriminating Weierstrass to see that the continuity of the sum could not be carried over from the binomial to the trigonometric series, because there was not the same kind of convergence in the latter case. If this surmise is correct, the discovery of uniform convergence in the case of the third discoverer also is closely connected with a Fourier series.

I have dwelt at some length on uniform convergence because its discovery marks both the culmination of the first and older epoch in the treatment of functional series, and the beginning of a new one. In uniform convergence and a study of the discontinuous we have sought for the chief springs of the modern *theory of functions of a real variable. By so doing we are led to assign as a fourth great service of Fourier's series the genesis of this theory.* It is not to be forgotten, however, that other sources have also copiously contributed. The morphology of one member of a body

must be in many ways perverted, if studied without correlation to the other members. But, after all, it is the Fourier series which gave the initial push and chief impetus to the construction of the function theory of the real variable.

This becomes still plainer if we take into consideration the comparatively recent point-set theory. Originally an off-shoot of the real function theory and still often treated by itself, it has been largely absorbed back into this theory, and its concepts already permeate analysis. Its founder was George Cantor, who was trained in the exact yet fertile school of Weierstrass. His earliest papers presaging this theory relate to a trigonometric series.

Two problems occupy his attention. The first is to show that if the series $\Sigma\,(a_n \sin nx + b_n \cos nx)$ is convergent throughout an entire interval, except possibly for a finite number of points, the coefficients a_n and b_n have for $n = \infty$ the limit 0. The second is to establish the uniqueness of the development of a function into a trigonometric series; in other words, to prove that when $\Sigma\,(a_n \sin nx + b_n \cos nx)$ is identically 0 over an interval, then each and every a_n and b_n must be 0. The requirement of convergence of the sum in the one case and of its vanishing in the other, was originally made for the entire interval, but Cantor found that it could be remitted for certain infinite aggregates of points without affecting the truth of the conclusions. He was led consequently to introduce the notion of the "*derivative of a point-set.*" Consider with him the set of points for which the requirement is omitted, and suppose that they cluster in infinite number in the vicinity of any point. This will be called a limit-point of the set. The totality of these limit-points is called the first derived set, or first derivative. This derived set of points may also have cluster points which form the second derivative; and so on. After introducing this concept, Cantor proved that the requirement could be remitted for any set of points whose nth derivative contains only a finite number of points and whose $(n+1)$th derivative accordingly vanishes.

In these very early papers of Cantor we have very clearly the beginning of his point-set theory. His attention is here concentrated upon an infinite aggregate of points, and the notion of the derived point-set was the first of the concepts by means of which he is able to distinguish between different infinite aggregates of points. Prior to Cantor no effort was made to distinguish qualitatively between them. To be sure, mathematicians were thoroughly conversant with the distinction between a continuous curve or set of points, on the one hand, and a merely dense aggregate of points such as the totality of points with rational coordinates. The raw material lay at hand for a beginning, especially in the work of Riemann and others on integration. Cantor alone saw the imperativeness of the need. In comparing infinite sets of objects and seeking a theory of the truly infinite he blazed a new path for the human mind. *As a fifth and a mighty influence of Fourier's series we have, therefore, to record the historic origin of the theory of infinite aggregates.*

Thus far in my sketch I have traced one strong, single current of influence of the Fourier's series. I have now to indicate some other effects without close relation to the foregoing.

In Fourier's "Analytical Theory of Heat" there are found what are said to be the first instances of the solution of an infinite number of linear equations with an infinite number of unknowns. He has, for example, to determine the coefficients in the equation:

$$1 = a \cos y + b \cos 3y + c \cos 5y + \cdots.$$

For this purpose he differentiates an even number of times, obtaining thus the system of equations

$$0 = a \cos y + b3^n \cos 3y + c.5^n \cos 5y + \ldots (n = 2, 4, \ldots)$$

Combining this with the preceding equation and putting $y = 0$, he obtains an infinite number of equations of first degree with an infinite number of unknowns, $a, b, c, \cdots$. To solve these he uses the first m equations to determine the first m unknowns, suppressing all the other unknowns, and finally determines their limiting values as m increases indefinitely. There is no time to point out the lack of rigor. Fourier uses his mathematics with the delightful freedom and naïveté of the physicist or astronomer who trusts in a mathematical providence.

This suggestive line of attack was not followed up, and indeed could not be, prior to the development of a theory of infinite determinants. When such a system of linear equations with an infinite number of unknowns came again to the foreground, the inciting cause was again a trigonometric series. I refer, as you know, to the work of our own astronomer, Hill. In his memoir on the "Motion of the Lunar Perigee" he had before him a differential equation of the following form, with numerical coefficients:

$$\frac{d^2w}{d\tau^2} = 2w\left({}_2 + \theta_1 \cos 2\tau + \theta_2 \cos 4\tau + \cdots\right).$$

Assuming a solution in the form

$$w = e^{ic\tau} \sum_{n=-\infty}^{n=+\infty} b_n e^{2in\tau}$$

(which except for the factor $e^{ic\tau}$ is only a trigonometric series under another guise), Hill obtains for the determination of c and the b_n an infinite system of equations linear in the b_n. The elimination of the b_n then gives c as the root of a certain infinite determinant, and then the values of the b_n are also found by use of infinite determinants.

The importance of Hill's results at once attracted the genius of Poincaré whose attention had, in fact, been previously drawn by Appell to an infinite system of linear equations. Poincaré now proceeded to consider the question of the convergence of infinite determinants, and in so doing laid a sound foundation for a new mathematical subject. In this new theory of infinite determinants the central thought is the passage, under restrictions to be properly ascertained, from a finite to an infinite system of linear equations. This principle here employed has been since applied in an even more striking manner by Fredholm, who was led through its use to his historic solution of a class of integral equations. In the theory of these equations the infinite determinant plays an indispensable rôle. *A sixth influence of Fourier's series is thus seen in the origin of a theory of infinite determinants,* also indirectly in the theory of integral equations for which it has supplied an important tool.

The seventh and the last influence on which I shall specifically dwell is more subtile, not so easily pointed out or demonstrated as some of the foregoing, but nevertheless one of the most far-reaching and probably the most pervasive of all. The physicist, astronomer, or mathematician has again and again to expand an arbitrary or assigned func[illegible]into a series of functions, the nature of which varies with the problem before one. When once the idea and method of expressing an arbitrary function in series of sines and cosines have been won, they can be extended to other series of functions, as for instance series of Bessel's functions, zonal harmonics, Lamé polynomials, spherical harmonics. For such developments the trigonometric series with its

applications has repeatedly served as a guide post. Numberless analogous results have been suggested thereby, though without definite statement of the fact. To take an example at random, the relation

$$\int_0^1 P_{2n}(x)P_{2m}(x)dx = 0 \qquad (m \neq n)$$

has its trigonometric analogues

$$\int_0^\pi \cos n\theta \cos m\theta\, d\theta = 0.$$

Who can deny, or who can affirm, in many such individual instances that the suggestion came from the trigonometric series? Yet in the bulk the debt is so great that he who runs can read it.

It is especially in connection with boundary value problems that we encounter series of functions. Now the trigonometric series was the inevitable tool for the first boundary value problems—those of vibrating strings, rods, columns of air, etc. Later, when Fourier crystallized the boundary value problems into classic shape, he used trigonometric series and, to lesser degree, similar series of Bessel's functions, obviously because these afforded him the simplest tools for the simplest problems. From series of sines of multiple angles he was led by certain problems in heat conduction to series of form $\Sigma c_i \sin a_i x$, where the a_i are roots of a certain transcendental equation. Thence the orientizing influence of Fourier's series is continued down to the modern development of normal functions in the theory of integral equations. All such influences are in the very warp and woof of mathematical development and can not be disentangled. To minimize or ignore them would be to give a distorted picture. They form a most vital and leading part of the mighty theory of harmonic and normal functions and of the boundary value theory.

The extent of these influences in the past gives rise naturally to the question of whether the trigonometric series will continue to exert such a moulding influence in the future. Certain results of Baire to be shortly mentioned incline one to answer negatively. Yet the questions regarding the convergence of the series and the character of the functions which it can represent are even to-day incompletely answered. When new implements are invented, it is still to these unanswered questions that the investigator naturally turns to test their worth, as, for example, Lebesgue with his great new concept of an integral which has application when Riemann's integral is void of sense, or Fejér with a method of summing a divergent series. Also the Fourier series still offers an occasional surprise. Who indeed would have anticipated Gibbs's discovery, since extended by Bôcher, which relates to the approximation curve $y=S_n(x)$, obtained by equating y to the sum of the first n terms of the series (2) above? As n increases indefinitely, the amount of the oscillation of the curve in the vicinity of each point of discontinuity of the limit does not tend toward the measure of the discontinuity, as would be supposed, but to this value increased in a certain definite ratio! But it may be reasonably expected that these surprises will become fewer and less important.

In this brief review I have neglected certain less analytic aspects, such as trigonometric interpolation and the use of the series in computation and in the perturbation theory. It has also not been necessary to emphasize the simplicity of structure of the series and its adaptation to computation. Neither do I need to speak of its correspondence in structure to so many periodic phenomena of nature, sound, light, the tides, etc. But I do wish, in closing, to emphasize and examine further, one aspect implied in all my preceding con-

siderations, the wonderful *pliability* of the series.

It was this pliability which was embodied in Fourier's intuition, commonly but falsely called a theorem, according to which the trigonometric series (I.) "*can express any function whatever between definite values of the variable.*" This familiar statement of Fourier's "theorem," taken from Thompson and Tait's "Natural Philosophy," is much too broad a one, but even with the limitations which must to-day be imposed upon the conclusion, its importance can still be most fittingly described as follows in their own words: The theorem "*is not only one of the most beautiful results of modern analysis, but may be said to furnish an indispensable instrument in the treatment of nearly recondite question in modern physics. To mention only sonorous vibrations, the propagation of electric signals along a telegraph wire, and the conduction of heat by the earth's crust, as subjects in their generality intractable without it, is to give but a feeble idea of its importance.*"

Truly, the theorem is so comprehensive in its mathematical content that we mathematicians may well query with one of my colleagues whether it may not have conditioned the form of physical thought itself—whether it has not actually forced the physicist often to think of complicated physical phenomena as made up of oscillatory or harmonic components, when they are not inherently so composed.

It is this same pliability of the series that has been a source of perpetual delight and surprise to the mathematician. It has revealed an undreamt-of power in analysis. It has stimulated intuition and vigor, and has helped to usher in a modern critical era in mathematics similar in spirit to the Greek period. It has separated differentiable from continuous functions; it has put the integral calculus on a basis of independence of the differential calculus; it has focused attention upon sets of irregularities and discontinuities whose study has started the point-set theory; it has opened the field of discontinuous functions to analysis and, above all, has engendered a theory of functions of the real variable.

To the mathematician the theory of analytic functions for some time appeared to be of much greater importance than the freaky theory of the real variable, because almost all the important functions of mathematics are analytic. Also, the same has been hastily assumed for physics because the real and imaginary components of an analytic function are harmonic functions satisfying Laplace's equation. But this is to ignore features of at least equal, if not of superior, importance. Not long ago many thought that the mathematical world was created out of analytic functions. It was the Fourier series which disclosed a *terra incognita* in a second hemisphere.

Here, in the new hemisphere, the mathematician has advanced beyond the boundary of the trigonometric series. It has been found that discontinuous functions representable through such series form a thoroughly restricted class. They belong to what Baire calls the first class of functions which are limits of convergent sequences or series of continuous functions, themselves of "class 0." These in turn may be used to generate new functions. Even as non-uniformly convergent Fourier series may give rise to discontinuous functions of Class 1, so non-uniformly convergent series of functions of this class may give a new sort of functions of Class 2, and so on. Indeed, to every transfinite number α of the first or second class there corresponds, as Lebesgue has shown, a definite class of functions. *Thus the Fourier series has, after all, a very limited range of representation*

in the totality of functions mathematically conceivable.

Even for functions of Class 0 or 1 the trigonometric series has a limited power of representation. This is manifest from an example given by Paul Du Bois Reymond of a continuous function which can not be represented by a trigonometric series. It remains to determine in the future just what properties are necessary and sufficient to characterize those functions of Classes 0 and 1 which are expressible by means of trigonometric series.

Earlier in my paper I pointed out that the generality of functions representable through Fourier's series was so great that the mathematician was led irresistibly to the Dirichlet definition of a function. If, namely, to every value of x in an interval we have a corresponding value of y, then y is called a function of x, no matter how the correspondence is set up, whether by a graph, a mathematical expression, a law, or any other way. To-day the pendulum has swung back to the old question of Euler. The study of representability in terms of trigonometric series has been succeeded by the broader question of the possibility of analytic expression in general. Now every continuous function, as is well known, can be represented by a uniformly convergent set of polynomials. Starting then from the totality of polynomials as a basis of functions for Class 0, we arrive successively at Baire's and Lebesgue's classes of functions corresponding to or, if you prefer, marked, by the transfinite numbers of the first and second classes.

Do these different classes of functions comprise all which are "*analytically expressible*"? Before answering the question it is necessary first to sharply define the phrase "analytically expressible." This is done by Lebesgue. Then, after broadening the content of these classes in a manner I have not the time to describe, he goes on to show that they do in truth comprise all such functions. The final question then confronts us: Are all possible functions included which are defined in accordance with the general definition of Dirichlet? In other words, are there functions *incapable of being "analytically expressed"*? Lebesgue by an example shows that this is the case. Our study of the Fourier series opened with the question: What is an arbitrary function? Here, at last, apparently, we have discovered the existence of a function of such a height or depth of arbitrariness as to be mathematically inexpressible. Having started with the Fourier series on a voyage of exploration, shall we conclude by saying that there is for us an unattainable pole?

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UNIVERSITY REGISTRATION STATISTICS

The registration returns for November 1, 1913, of thirty of the leading universities of the country will be found tabulated on the following page. Specific attention should be called once again to the fact that these universities are neither the thirty largest universities in the country, nor necessarily the leading institutions. The only universities which show a decrease in the grand total attendance (including the summer sessions) are Harvard, Western Reserve and Yale, the attendance of the two institutions last named having remained practically stationary. The largest gains in terms of student units, including the summer attendance, but making due allowance by deduction for the summer session students who returned for instruction in the fall, were registered by New York University (965), Illinois (944), Columbia (927), Wisconsin (749), Pennsylvania (681), California (614), Iowa (598),

Ohio State (503), Chicago (483), and Michigan (388). Last year there were only five institutions that showed a gain of over 300 students, namely, Columbia, California, New York University, Texas and Harvard. Omitting the summer session attendance, the largest gains this year have been made by Illinois (887), New York University (772), California (640), Pennsylvania (571), Iowa (538), Wisconsin (493), Ohio State (434), Michigan (381), Chicago (353), Syracuse (305), Washington University (267), and Columbia (255). It will thus be seen that this year twelve institutions exhibited an increase of over 200 students in the fall attendance, as against eight in 1912 and four in 1911. Of these institutions eight are in the west and four in the east.

According to the figures for 1913, the thirty institutions, inclusive of the summer session rank as follows: Columbia (9,929), California (7,071), Chicago (6,834), Michigan (6,008), Pennsylvania (5,968), Wisconsin (5,890), Harvard (5,627), Cornell (5,612), New York University (5,508), Illinois (5,259), Ohio State (4,111), Minnesota (3,932), Northwestern (3,877), Syracuse (3,845), Yale (3,263), Missouri (3,135), Texas (3,106), Nebraska (2,850), Kansas (2,610), Iowa (2,542), Tulane (2,298), Indiana (2,271), Pittsburgh (1,906), Cincinnati (1,871), Stanford (1,756), Princeton (1,599), Western Reserve (1,370), Johns Hopkins (1,311), Washington University (1,225), and Virginia (885), whereas last year the order was Columbia, California, Chicago, Harvard, Michigan, Cornell, Wisconsin, Minnesota, Pennsylvania, New York University, Illinois, Northwestern, Ohio State, Syracuse, Yale, Texas, Missouri, Nebraska, Kansas, Tulane, Iowa, Cincinnati, Pittsburgh, Stanford, Princeton, Western Reserve, Washington University, Johns Hopkins, Virginia. If the summer-session enrollment be omitted, the universities in the table rank in size as follows: Columbia (6,403), Pennsylvania (5,305), Michigan (5,304), California (5,225), Harvard (4,922), Illinois (4,835) and New York University (4,835), Cornell (4,760), Wisconsin (4,450), Northwestern (3,776), Chicago (3,719), Ohio State (3,708), Syracuse (3,699), Minnesota (3,616), Yale (3,263), Missouri (2,547), Nebraska (2,482), Texas (2,373), Kansas (2,308), Iowa (2,294), Pittsburgh (1,906), Cincinnati (1,871), Stanford (1,743), Princeton (1,599), Indiana (1,417), Western Reserve (1,370), Tulane (1,244), Washington University (1,225), Johns Hopkins (1,012) and Virginia (885), whereas last year the order was: Columbia, Michigan, Harvard, Cornell, California, Pennsylvania, New York University, Wisconsin, Illinois, Northwestern, Minnesota, Syracuse, Chicago, Ohio State, Yale, Nebraska, Missouri, Texas, Kansas, Cincinnati, Pittsburgh, Iowa, Stanford, Princeton, Western Reserve, Tulane, Washington University, Virginia, and Johns Hopkins.

Including the summer-session attendance the largest gains in the decade from 1903 to 1913 were made by Columbia (5,372), California (3,594), New York University (3,331), Pennsylvania (3,324), Chicago (2,688), Wisconsin (2,669), Ohio State (2,423), Cornell (2,174), Michigan (2,082) and Illinois (2,020).

So far as the individual faculties of the various universities are concerned, Harvard with 2,350 men and 564 women (Radcliffe College) leads in the number of college undergraduates, being followed by California, with 1,112 men and 1,626 women; Michigan, with 1,736 men and 784 women; Stanford, with 1,243 men and 500 women; Chicago, with 936 men and 767 women; Kansas, with 942 men and 688

	California	Chicago	Cincinnati	Columbia	Cornell	Harvard	Illinois	Indiana	Iowa	Johns Hopkins	Kansas	Michigan	Minnesota	Missouri
College, Men	1112	936	200	841	} 1120	2350	427	694	610	178	942	1736	639	834
College, Women	1626	767	419	623		564	402	423	553	....	688	784	882	516
Scientific Schools*	828	....	406	665	1343	76	1001	248	221	79	372	1282	638	360
Law	142	211	....	450	267	695	106	85	201	....	174	553	193	103
Medicine	119	191	84	341	107	306	445	132	112	368	107	353	172	75
Non-professional graduate schools	408	480	170	1496	315	489	268	83	130	219	99	225	127	123
Agriculture	530	....	....	....	1354	....	792	....	....	....	....	....	464	501
Architecture	†	....	....	143	143	61	351	....	....	....	23	120	43	....
Art	†	....	....	....	....	....	†	58	42	....	20	†	....	....
Commerce	280	160	97	....	....	113	282	....	....	....	....	†	....	....
Dentistry	90	....	....	....	....	203	94	....	244	....	....	282	268	....
Divinity	....	125	....	....	....	56	....	....	....	....	....	....	....	....
Forestry	†	....	....	....	....	9	†	....	....	....	....	†	41	†
Journalism	....	....	....	106	....	....	†	73	....	....	....	†	....	64
Music	†	....	....	16	....	....	78	93	87	....	130	†	25	....
Pedagogy	†	287	240	1670	....	....	....	77	†	....	181	†	87	182
Pharmacy	114	....	....	441	....	....	255	....	57	....	61	96	95	....
Veterinary Medicine	†	....	....	....	122	....	†	....	....	....	....	....	....	....
Other Courses	....	808	486	....	....	....	350	....	86	168	....	....	10	....
Deduct Double Registration	24	246	231	389	11	....	16	549	49	....	489	127	68	211
Total	5225	3719	1871	6403	4760	4922	4835	1417	2294	1012	2308	5304	3616	2547
Summer Session 1913	2363	3771	....	4539	1392	797	713	1084	426	347	510	1408	531	810
Deduct Double Registration	517	656	....	1013	540	92	289	230	178	48	208	704	215	222
Grand Total 1913, Nov. 1	7071	6834	1871	9929	5612	5627	5259	2271	2542	1311	2610	6008	3932	3135
Grand Total 1912, Nov. 1	6457	6351	1924	9002	5412	5729	4315	2192	1944	1058	2403	5620	3737	2871
Grand Total 1910, Nov. 1	4552	5883	1416	7411	5169	5329	4659	2132	1957	890	2246	5339	4972	2678
Grand Total 1908, Nov. 1	3644	5114	1364	5677	4700	5342	4400	2113	2356	707	2086	5188	4607	2558
Grand Total 1903, Nov. 1	3477	4146	1068	4557	3438	6013	3239	1614	1260	694	1319	3926	3550	1540
Extension and Similar Courses	638	3182	....	3644	597	1100	....	242	....	....	1185	235	1546	152
Officers	525	337	261	907	725	731	737	109	275	208	200	537	465	305

	Nebraska	New York	Northwestern	Ohio State	Pennsylvania	Pittsburgh	Princeton	Stanford	Syracuse	Texas	Tulane	Virginia	Washington University	Western Reserve	Wisconsin	Yale
College, Men	541	486	479	485	} 409	305	1267	1243	} 1340	811	166	396	152	459	828	1402
College, Women	733	221	609	412		87	....	500		709	152	....	229	358	776	....
Scientific Schools*	306	233	80	811	657	266	156	....	308	288	164	101	169	....	775	1133
Law	200	771	294	181	381	143	....	140	291	339	94	257	80	109	161	130
Medicine	102	432	178	....	283	123	....	64	96	170	399	102	60	164	79	49
Non-professional graduate schools	169	369	85	121	438	46	176	134	75	77	27	43	57	9	290	334
Agriculture	423	....	....	889	....	....	....	....	70	38	....	....	....	....	968	....
Architecture	1	....	....	65	259	....	....	....	50	54	25	....	46	....	....	....
Art	64	....	46	....	....	....	....	....	178	....	85	....	305	....	....	53
Commerce	22	2013	567	....	1430	....	....	....	....	57	....	....	....	....	374	....
Dentistry	....	....	566	....	589	209	....	....	....	....	55	....	111	139	....	....
Divinity	....	....	233	....	....	....	....	....	....	....	....	....	....	....	....	100
Forestry	36	....	....	65	....	....	....	....	220	....	....	....	....	....	12	32
Journalism	....	†	39	....	....	....	....	....	....	....	21	....	....	....	91	....
Music	....	....	407	....	26	....	....	....	925	....	73	....	....	....	81	85
Pedagogy	164	420	122	166	699	421	....	....	25	443	91	....	....	....	42	....
Pharmacy	23	....	135	92	....	209	....	....	....	63	7	....	....	124	40	....
Veterinary Medicine	....	11	....	163	125	....	....	....	....	....	....	....	....	....	....	....
Other Courses	....	....	296	280	9	157	....	....	233	....	....	....	48	45	45	....
Deduct Double Registration	302	121	360	22	....	60	....	338	112	676	115	14	32	37	112	55
Total	2482	4835	3776	3708	5305	1906	1599	1743	3699	2373	1244	885	1225	1370	4450	3263
Summer Session 1913	511	933	193	703	865	....	....	38	260	981	1163	‡	....	....	2120	....
Deduct Double Registration	143	260	92	300	202	....	....	25	114	248	109	....	....	....	680	....
Grand Total 1913, Nov. 1	2850	5508	3877	4111	5968	1906	1599	1756	3845	3106	2298	885	1225	1370	5890	3263
Grand Total 1912, Nov. 1	2811	4543	3632	3608	5287	1833	1568	1670	3529	3016	2249	799	958	1378	5141	3265
Grand Total 1910, Nov. 1	2733	3947	3543	3181	5187	....	1451	1648	3248	2597	1985	688	796	1274	4745	3287
Grand Total 1908, Nov. 1	3154	3951	3113	2700	4555	....	1314	1541	3204	2410	1171	757	806	1016	3876	3466
Grand Total 1903, Nov. 1	2513	2177	2740	1688	2644	....	1434	1370	2207	1309	1037	613	761	765	3221	2990
Extension and Similar Courses	902	1676	....	227	379	411	....	....	....	757	117	....	319	....	2907	....
Officers	354	425	434	291	564	290	217	218	280	....	313	101	203	252	633	571

Note.—The grand totals of the University of Minnesota prior to 1912 are inclusive of "extension and similar students," which were listed separately in 1912 and 1913, hence the apparent decrease.

* Includes schools of mines, engineering, chemistry and related subjects.

† Included elsewhere.

‡ 1,220 students in attendance on summer courses.

women; Wisconsin, with 828 men and 776 women; Minnesota, with 639 men and 882 women; Texas, with 811 men and 709 women; Columbia, with 841 men and 623 women; Yale, with 1,402 men; Nebraska, with 541 men and 733 women, and Princeton, with 1,267 men.

In agriculture, Cornell leads with 1,354 students, being followed by Wisconsin with 968, Ohio State with 889, and Illinois with 792. In architecture Illinois with 351 is followed by Pennsylvania with 259, and Columbia and Cornell with 143 each. Washington University, with 305 art students, leads in that field, being followed by Syracuse with 178; while New York University continues to lead in commerce with 2,013 students, being followed by Pennsylvania with 1,430, Northwestern with 567, Wisconsin with 374, Illinois with 282, and California with 280. The largest dental school is at Pennsylvania, where 589 students are enrolled, as compared with 566 at Northwestern, 282 at Michigan, and 268 at Minnesota. Northwestern has the largest divinity school, enrolling 233 students, as against 125 at Chicago, 100 at Yale and 56 at Harvard; these are the only universities in the list that maintain schools of theology.

Syracuse has 220 students of forestry, Ohio State 65, Minnesota 41, Nebraska 36, and Yale 32; at California, Illinois, Michigan and Missouri, the forestry students are counted in with other departments. Columbia has a long lead in the number of non-professional graduate students, there being no less than 1,496 students enrolled in its faculties of political science, philosophy and pure science. Columbia is followed by Harvard with 489 students, Chicago with 480, Pennsylvania with 438, and California with 408. Columbia has the largest school of journalism, enrolling 106 students as compared with Wisconsin's 91, Indiana's 73, and Missouri's 64. The largest law school is at New York University, where 771 students are registered in this subject; Harvard follows with 695 students, Michigan with 553, and Columbia with 450. In medicine Illinois leads with 445, being followed by New York University with 432, Tulane with 399, Johns Hopkins with 368, Michigan with 353, Columbia with 341, and Harvard with 306. Syracuse has the largest number of music students, namely, 925, there being 407 at Northwestern and 130 at Kansas. The Teachers College of Columbia University is by far the largest school of education connected with any of the institutions in the list. It has an enrollment this fall of no less than 1670 students, as against 699 at Pennsylvania, 443 at Texas, 421 at Pittsburgh and 420 at New York University. Columbia also has by far the largest school of pharmacy, enrolling 441 students, as against 255 at Illinois, 209 at Pittsburgh, and 135 at Northwestern. As for the scientific schools, Cornell continues to maintain its lead in this branch, enrolling 1,343 students, as against Michigan's 1,282, Yale's 1,133, Illinois's 1,001, California's 828, Ohio State's 811, Wisconsin's 775, Columbia's 665, Pennsylvania's 657, and Minnesota's 638. In veterinary medicine Ohio State leads with 163, being followed by Pennsylvania with 125, and Cornell with 122. All of the above figures for individual faculties are exclusive of the summer-session attendance. The largest summer session in 1913 was at Columbia University, where 4,539 students were enrolled, as against 3,771 at Chicago, 2,363 at California, 2,120 at Wisconsin, 1,408 at Michigan, 1,392 at Cornell, 1,163 at Tulane, and 1,084 at Indiana.

The largest number of officers is found at Columbia, where the staff of teaching and administrative officers consists of 907 mem-

bers, as against 737 at Illinois, 731 at Harvard, 725 at Cornell, and 633 at Wisconsin.

The 638 students enrolled at the University of California in extension and similar courses are distributed as follows: San Francisco Institute of Art 201, Wilmerding School of Industrial Arts 155, University Farm School 187, and short courses in agriculture 95.

The 3,182 students listed under extension and similar courses at the University of Chicago are enrolled in correspondence study courses. The 808 students mentioned under other courses are enrolled in regular university courses given primarily for teachers which meet on Saturday mornings and late in the afternoons.

Among the 84 students of medicine enrolled at the University of Cincinnati, 23 are registered in clinics and pathological work. The 486 students mentioned under other courses are enrolled in evening academic courses.

Of the 3,644 students enrolled under extension and similar courses at Columbia University, 1,152 are students in special classes at Teachers College and 2,492 are students in extension courses.

Of the 1,120 students registered at Cornell University under arts and sciences, 950 are candidates for the degree of A.B. and 170 for the degree of B.Chem. Of the total number of 4,760 students enrolled in the fall, 4,273 are men and 487 women. The students mentioned under extension and similar courses were enrolled in the short winter course in agriculture for 1912–13.

The 113 students mentioned under commerce at Harvard University are enrolled in the graduate school of business administration. Of the 61 students in architecture, 23 are enrolled in landscape architecture, while the 76 students mentioned under scientific schools are enrolled in the graduate school of applied science. Only one department, forestry, which is a part of the graduate school of applied science, shows a marked decrease, the enrollment having dropped from 18 to 9. This may possibly be accounted for by the fact that this year for the first time students were expected to register in July, instead of in September, and this may not have been sufficiently widely known. Also, the course has been changed somewhat, including general work in the first year and special work in the second year. In landscape architecture, which is also in the graduate school of applied science, there has been a marked increase, 23 students being enrolled this year, as opposed to 15 in 1912, and 7 in 1911.

Of the 350 students given under other courses at the University of Illinois, 310 are enrolled in household science and 40 in library economy. In Chicago, the departments of medicine and dentistry have been reopened with higher entrance requirements—hence the material loss in the registration in these schools as compared with the registration of two years ago.

The students mentioned under scientific schools at Indiana University are pursuing courses in chemistry, provision for technical work being made at Purdue University.

The 86 students given under other courses at the State University of Iowa are enrolled in courses for nurses.

At Johns Hopkins, 169 men and 39 women are enrolled in the graduate school, and 15 are taking graduate work in engineering. Of the 368 students given under medicine, 8 are physicians attending special courses.

Of the 353 students given under medicine at the University of Michigan, 75 are enrolled in the homeopathic medical college.

At the University of Minnesota, agricul-

ture, inclusive of the home economic division, shows a steady growth. The apparent increase in the graduate school is explained by regulations concerning registration rather than by an influx of advanced students. However, impetus has been given to this department by the appointment of Professor Guy S. Ford, formerly of the University of Illinois, as dean. The law school is feeling the full effect of the requirement of two years of academic work for admission to regular courses, also the effect of discontinuing night classes. Many of the subjects heretofore offered by the law school in evening courses have been transferred to the extension department. Heretofore little emphasis has been given to music. This year marks the beginning of a regular four years' course in arts and music leading to the degree of bachelor of arts in music, consequently these students are listed separately this year for the first time. The requirements for admission are the same as for the college of science, literature and the arts. The college of engineering has secured F. E. Mann, of the University of Illinois, to take charge of the department of architecture, and the courses in architecture and architectural engineering have now been permanently established. The requirements for admission are the same as the engineering courses and the B.S. and appropriate professional degrees are granted at the close of four and five years, respectively. The reorganization of the extension division and the advent of Professor R. R. Price, formerly of Kansas University, as director of this department, explain the increased enrollment in this division.

The decrease in enrollment in the school of law at the University of Missouri is due to the fact that during this session all of the three classes in that school are based upon an admission requirement of two years of college work, while the third-year class of last year was admitted under the former requirement of four years of high school work. There is a decrease in the school of engineering due to a similar reason, but as there has been an increase in the school of mines, the total enrollment in technical schools shows a slight increase. It has been anticipated that a similar decrease would be manifested in the school of journalism. Notwithstanding the fact, however, that there is during the present session only one of the classes which entered under the former requirements as compared with two such classes during the session of 1912–13, the total enrollment in the school shows an increase.

At the University of Nebraska, there is but little change in registration this year as against 1912, the most apparent increase being in the school of agriculture, at the expense of the professional schools.

Every school in New York University with the exception of the veterinary college shows a marked increase. The increase in the college of arts and pure science is due largely to the transfer of the medical preparatory class from the collegiate division to the freshman class of the college.

Of the 296 students enrolled in other courses at Northwestern University, 172 are registered in the school of oratory and 124 are taking courses for nurses. There have been noteworthy increases in attendance in several of the schools of the university, especially in the college of engineering, the dental school, and the school of commerce. There is an increase in the college of liberal arts in spite of the increased tuition fees. The same increase in fees was put into effect in the college of engineering, but, nevertheless, the enrollment is the most encouraging in the history of the college, it being due to a consistent and dignified sys-

tem of publicity. In the dental school, the university is increasingly strict in administering entrance requirements, and yet the enrollment has increased at a phenomenal rate. The increase in the school of commerce is an evidence of the need of such work in a large city; these courses are given in the evening, and the registration is in great part made up of men who are in business during the day. In the statistics of November 1, 1912, the report showed 368 students enrolled in the law school; the figure should be 268.

The 280 students given by Ohio State University under other courses are enrolled in home economics. The 227 students in extension and similar courses represent the enrollment of 1912–13. The fall enrollment shows substantial increases in all colleges except the college of law, which is 13 short of the enrollment November 1, 1912. The largest gains in the colleges have been as follows: agriculture 188, engineering 109, arts 94, and education 37. The increase in the fall enrollment is 434. These increases are probably due to a revival of interest among the alumni of the university. Two years ago, under the able leadership of Mr. Ralph D. Mershon, of New York, the alumni and former students of the university were reorganized into a live association. Local organizations have been formed in all parts of the state and in all prominent cities of the country. A permanent secretary, who gives all of his time to alumni interests, has been employed. The annual observance of "Ohio State Day" by alumni and former students throughout the state and country has done much to bring the university into prominence in local communities. A new department of competitive and recreative athletics was created by the board of trustees last June. The director of athletics and the men associated with him have been given faculty rank. A limited amount of credit will be given for work in courses in the teaching of athletics. A new course in applied entomology, leading to the degree of bachelor of science in entomology, has been added to the curriculum. Two new combination courses have also been arranged, arts—agriculture and arts—home economics, making it possible for a student to receive two degrees in five years. The last General Assembly of Ohio authorized the establishment of an engineering experiment station at the university and the organization of a college of medicine. No definite action has been taken by the board of trustees. The General Assembly also authorized and directed the university to establish and organize a university extension division, for the purpose of carrying on educational extension and correspondence instruction throughout the state.

At the University of Pennsylvania, the 379 students listed under extension and similar courses are enrolled in courses at Wilkes Barre (185) and Scranton (194). The 9 students listed under other courses are enrolled in hygiene. The 699 students listed under pedagogy are enrolled in college courses for teachers. Nearly all departments share in the gains in enrollment, the most noticeable increase being in the Wharton school of finance and commerce, and in the evening school of accounts and finance. The school of architecture continues its advance in registration, the total this year being 259, or an increase of 43. The graduate school, with a total enrollment of 438, is 67 ahead of the preceding year. The law school, with a total enrollment of 381, is just 2 short of the figures of last year. The medical school is still feeling the effects of the raise in entrance requirements put into effect within the past few years. The enrollment of 283 shows a loss of 37. The dental school, with 589

students, has an increase of 80. Other departments show slight gains or losses which are negligible.

At the University of Pittsburgh, the 157 students listed under other courses are male candidates for degrees in economics. Of the 411 students given under extension and similar courses, 127 are taking teachers' courses without credit and 284 are enrolled in the evening school.

The students at Leland Stanford University listed under law and medicine and the graduate school are all included also under the college.

At Syracuse University, of the 233 students listed under separate courses, 50 are taking work in library economy; 275 students from the college of liberal arts are also taking work in education.

At Tulane University, 7 are enrolled in tropical medicine and hygiene and 22 are taking post-graduate medical work. The students mentioned under scientific schools are enrolled for engineering and sugar chemistry. The increase in numbers of the college of technology is due to changes made in the college, amounting practically to reorganization.

The 48 students listed under other courses at Washington University are in the school of social economy. The 319 students listed under extension and similar courses are taking Saturday courses for teachers and others in the college.

The 45 students listed under other courses at Western Reserve University are taking work in library economy. In Adelbert College, the college for women, the graduate school, the school of law and the library school, the annual tuition fee has been advanced from $100 to $125, and in the school of medicine from $125 to $150, these charges going into effect this year. The loss in enrollment in the school of law is due to the graduation of the last class received upon the non-graduate basis, all classes now being upon the college graduate basis. The change from the non-graduate to the graduate basis is seen to have taken effect with but a small loss in numbers. The loss in enrollment in the school of medicine is more apparent than real, the difference being caused largely by the recent yearly graduation of two classes,—Ohio Wesleyan University and Western Reserve University,—and the reception of one class on the Western Reserve University college graduate basis only.

Of the 45 students listed under other courses at the University of Wisconsin, 43 are enrolled in library economy and 2 are taking work in public health. The 40 students in pharmacy include 24 students in a two-year pharmacy course, for which four years of high school preparation is not required. A forest rangers' course consisting of 12 students began work at the university January 7, 1913. These students did not complete their field work until November 30.

The most considerable increase in enrollment at Yale University is in the college. The entering classes in the law and medical schools also show noticeable increases and indicate that the requirement of a college degree for admission to the law school and of two years of college work for admission to the medical school, adopted two years ago, is being appreciated by the best grade of students. The only noticeable falling off is in the graduate school. This is due exclusively to the new requirement of two years' work for the master of arts degree instead of one year, as in force at Yale prior to 1912 and still the custom at almost all the American universities.

RUDOLF TOMBO, JR.

COLUMBIA UNIVERSITY

THE MASSACHUSETTS INSTITUTE OF TECHNOLOGY AND HARVARD UNIVERSITY[1]

In this agreement, "the institute" means the Massachusetts Institute of Technology, and "the university" means Harvard University. It is understood that any action of the president and fellows of Harvard College shall require the consent of the board of overseers wherever such consent is necessary under the laws governing the university.

I. The university and the institute shall be unaffected in name, organization, title to and rights over property, or in any other way not specifically mentioned in this agreement.

II. The university and the institute shall cooperate in the conduct of courses leading to degrees in mechanical, electrical, civil and sanitary engineering, mining and metallurgy, and in the promotion of research in those branches of applied science. The courses and research shall be conducted in accordance with the provisions of this agreement and on the site in Cambridge recently acquired by the institute bordering on Massachusetts Avenue and the Charles River Embankment or on any other site that may be agreed upon should future conditions render an extension or change of site desirable.

III. Subject to the reservations hereinafter set forth the university shall devote to the purposes referred to in Section II. the net income of all funds that are credited on its books to the Lawrence Scientific School; also the use of all machinery, instruments, and equipment that are suited to these purposes and that the university does not in its opinion need more urgently for other purposes; also not less than three fifths of the net income of the Gordon McKay Endowment; also the income of all property that it may acquire hereafter for the promotion of education or research in the branches of applied science referred to in Section II.; also such further sums as it may from time to time feel able to contribute.

IV. Subject to the reservations hereinafter set forth, the institute shall devote to the purposes referred to in Section II. all funds, or the income of all funds, that it now holds or hereafter acquires for the promotion of education or research in the branches of applied science mentioned in that section, and in addition to this as much of the funds, or income of funds, that it holds for general purposes as is not in its opinion more urgently required for other purposes.

V. Students' fees for courses in the branches of applied science mentioned in Section II., shall be devoted to the purposes referred to in that section. These fees shall for the first ten years be deemed to be contributed by the two institutions in the proportion of the numbers of the students following these courses in the institute and in the university's graduate schools of applied science, respectively, during the year 1913–14. At the end of ten years a different arrangement shall be made, if, in the opinion of the two corporations, it appears to be more equitable. The fees of students pursuing courses in the subjects referred to in Section II. in the university's graduate schools of applied science at the time when this agreement is adopted shall be unaffected by any change brought about by this agreement. For all other students the amount of the fees for complete courses leading to those degrees of the institute and of the university that are granted through the operation of this agreement shall be $250 per annum until changed by agreement between the two corporations. The amount of fees for partial courses and for research shall be determined as may be agreed upon from time to time.

VI. The funds available for education and research in the branches of applied science referred to in Section II. shall be expended through the bursary of the institute in the payment of salaries, the maintenance of scholarships, the care of grounds, and the erection and maintenance of buildings and equipment or otherwise as may be agreed upon from time to time, it being expressly provided that all proposed appropriations shall be approved by the corporation that supplies the funds, and that buildings shall be erected only

[1] Agreement ratified by the corporations of both institutions on January 6.

from the share of the funds supplied by the institute.

VII. All members of the instructing staff in the departments of mechanical, electrical, civil and sanitary engineering, mining and metallurgy, who give instruction in courses leading to the degrees both of the university and of the institute, shall be appointed and removed by the corporation that pays their salaries after consultation with the other corporation.

VIII. All students registered at the institute in the various numbered professional courses covered by Section II. that lead to degrees of the university shall be deemed to be prospective candidates for such degrees unless they signify a contrary intention, and shall be entitled to the same rights and privileges as students in the professional schools of the university.

IX. The president or acting president of the institute shall be the executive head for all the work carried on under this agreement. As an evidence of his responsibility in directing it he shall make an annual report to both corporations. When any future president or acting president is to be selected, the president or acting president of the university shall be invited to sit with the committee that recommends the appointment of a president or acting president to the corporation of the institute.

X. As soon as this agreement goes into effect, the faculty of the institute shall be enlarged by the addition thereto of the professors, associate professors and assistant professors of mechanical, electrical, civil and sanitary engineering, mining and metallurgy, in the university's schools of applied science. These persons shall acquire the titles and privileges of the same rank in the institute while retaining their titles and privileges in Harvard University, and the terms and conditions of their employment and their salaries shall be unaffected by the change. The professors, associate professors and assistant professors of the institute in the departments of mechanical, electrical, civil and sanitary engineering, mining and metallurgy, shall acquire the titles and privileges of the same rank in Harvard University while retaining their titles and privileges in the institute, and the terms and conditions of their employment and their salaries shall be unaffected by the change. All professors, associate professors and assistant professors appointed under the operation of Section VII. shall have the titles and privileges of professors of the university and of the institute, including the right to benefit from the pension system of both institutions.

Additions to the faculty of the institute shall be made by the appointment of professors, associate professors or assistant professors, under the operation of Section VII., or by the corporation of the institute for other purposes. The faculty constituted as indicated above shall, subject to such directions as may be given by the corporation of the institute, prescribe the courses and conditions of entrance thereto leading to all degrees granted by the institute. The same faculty shall, subject to such directions as may be given by the corporation of the university, prescribe the courses and conditions of entrance thereto leading to all degrees granted by the university under the operation of this agreement.

XI. Degrees shall be conferred by the institute and by the university acting separately on the recommendation of the faculty referred to in Section X.

XII. It is expressly provided that, as regards the funds and property of the university and of the institute respectively referred to in Sections III. and IV., this agreement shall be subject to any special terms and requirements upon which such funds and property may be held; and any property or funds that may be held at any time by either corporation under such terms and restrictions as would prevent their use precisely as is indicated in this agreement, shall, nevertheless, be used by the two corporations respectively for the support, benefit or encouragement of a cooperative effort in the field of education and research in engineering and mining in such manner as may be permissible or in accordance with the trusts upon which they may be held.

XIII. Whereas, doubts might arise as to

the legal effect of an omission from this agreement of any provision for its termination, it is hereby provided that the agreement may be terminated either by the university or by the institute, but that no termination shall be made except upon notice from one party to the other of at least five years unless a shorter time be mutually agreed upon.

XIV. This agreement shall take effect when finally adopted and approved by the corporation and board of overseers of the university and the corporation of the institute; and the cooperation referred to in Section II. shall begin when the institute is ready to open courses in engineering and mining on the site in Cambridge mentioned in that section.

STATEMENT BY PRESIDENT LOWELL

Friends of Harvard University and the Massachusetts Institute of Technology—and they have many friends in common—have long deplored the rivalry of two schools of engineering competing on opposite sides of a river. The disadvantages have been made even more evident by the decision of the institute to cross the Charles; but the difficulty of making an arrangement satisfactory to both parties has hitherto been very great, and in fact the obstacles to a combination between rival institutions supported by and serving the same community have been one of the grave defects of higher education in America. This difficulty seems at last to have been overcome here by a plan for cooperation in the conduct of one school of engineering and mining. The plan is favorable to both institutions. Both gain thereby. Which gains the most can probably not be determined, and certainly has not been computed, for the leading motive with the authors of the agreement has lain in another plane. Both institutions exist for the promotion of instruction and research. Each is a means to an end larger than itself, the welfare of the community as a whole; and that both acting in concert can further this end better than either working alone can not be doubted. By the combination of resources and momentum a school ought to be maintained unequalled on this continent and perhaps in the old world.

A. Lawrence Lowell

STATEMENT BY PRESIDENT MACLAURIN

The advantages of cooperation between the institute and Harvard have long been the subject of discussion. With reference to the present plan of cooperation, I beg to make the following statements:

1. The Honorable Richard Olney, in a carefully considered legal opinion, says "Cooperation between educational institutions for a legitimate purpose common to both is certainly not illegal and in this case wholly desirable. The 'agreement' seems to me to spell cooperation and nothing more, involves no merger of corporations or their property interests, and can be carried into effect without violation of charters or of the trusts upon which funds are held."

2. Its adoption by the corporation is unanimously recommended by the executive committee.

3. It is approved by each of the last five presidents of the Alumni Association, and by the president-elect. The president of the Alumni Association, Mr. Frederic H. Fay, writes: "I heartily commend this effort on your part, and I believe that when it is found that an agreement, such as you have proposed, can be carried out to the satisfaction of the authorities of both institutions, you will find that you have the great body of Tech Alumni behind you, and that you will have added to the prestige, and usefulness and strength of the institute."

4. It is approved by all the heads of the institute's departments that are directly affected and by all the other senior members of the faculty who have been consulted with reference to it.

5. It leaves the institute so entirely independent that it can appoint any officer or instructor that it pleases, it can appropriate its funds as it pleases, and it can make any regulations that it pleases with reference to the courses leading to its degree.

I believe that the adoption of this agreement is a forward step of very great import to the

future of education in this country. Incidentally, it would be of great advantage to each of the cooperating institutions, but it would be especially significant in the emphasis that it would give to the fact, so often overlooked, that educational institutions do not exist for themselves and that their sole duty is to make the best provision that can possibly be made for those who are rising to manhood and for their successors. Under the scheme of cooperation here proposed, it would be possible to maintain a much stronger school of applied science than either institution alone could furnish, and it would be possible to keep that school practically unrivalled in America—and indeed, in the world.

SCIENTIFIC NOTES AND NEWS

DR. BENJAMIN OSGOOD PEIRCE, since 1888 Hollis professor of mathematics and natural philosophy in Harvard University, died from disease of the heart at his home in Cambridge on January 14.

DR. HERMAN M. BIGGS has been appointed by Governor Glynn to be commissioner of health for the state of New York.

DR. ERNEST RUTHERFORD, Langworthy professor of physics in the University of Manchester, has been made a knight.

IN the Academy of Sciences of St. Petersburg, Sir William Ramsay has been advanced from a corresponding to an honorary member.

AT the annual meeting of the Federation of American Societies for Experimental Biology, an organization which includes the Physiological Society, the Society of Biological Chemistry and the Society for Pharmacological and Experimental Therapeutics, held in Philadelphia, the following officers were elected: *President,* Dr. Graham Lusk, New York City; *Vice-president,* Dr. Carl Alsberg, Washington, D. C.; *Secretary,* Dr. P. A. Shaffer, St. Louis; *Treasurer,* Dr. D. D. Van Slyke, New York City, and councilors, Professor J. J. Abel, Baltimore, and Professor A. B. Macallum, New York, and Dr. T. B. Osborne, New Haven, Conn.

PROFESSOR W. B. CANNON, of Harvard University, was elected president of the American Physiological Society at the meeting in Philadelphia.

AT the thirtieth session of the American Association of Anatomists held in Philadelphia at the University of Pennsylvania the following officers were elected: *President,* Professor G. Carl Huber, University of Michigan; *Vice-president,* Professor Frederic T. Lewis, Harvard Medical School; *Secretary-treasurer,* Professor Charles R. Stockard, Cornell Medical College.

AT the meeting of the American Phytopathological Society recently held at Atlanta the following officers were elected: *President,* Dr. Haven Metcalf, Washington, D. C.; *Vice-president,* Dr. Frank D. Kern, State College, Pa.; *Counsellor,* Professor H. R. Fulton, West Raleigh, N. C.

A DINNER in honor of Dr. Livingston Farrand, professor of anthropology in Columbia University, who has accepted the presidency of the University of Colorado, was held by his colleagues at the Faculty Club, Columbia University, on January 13.

THE following new appointments of members of the gardening staff at Kew are quoted in *Nature* from the *Kew Bulletin:* Mr. G. S. Crouch, to be assistant director of horticulture in the Egyptian department of agriculture; Mr. T. H. Parsons, to be curator of the Royal Botanic Gardens, Peradeniya, Ceylon, in succession to Mr. H. F. Macmillan, who has been appointed superintendent of horticulture in the department of agriculture, Ceylon; Mr. C. E. F. Allen, to be curator of the Botanic Garden, Port Darwin, Northern Territory, South Australia, in succession to Mr. N. Holtze, deceased.

MRS. AGNES CHASE, assistant in systematic agrostology, U. S. Department of Agriculture, has returned from Porto Rico where she has been collecting and studying grasses for about two months. Of the 123 species of grasses known from the island she obtained all but three, and about 40 additional species. *Arthrostylidium sarmentosum* Pilger, a climbing

bamboo, known only in the sterile condition, was obtained in flower.

An alumni chapter of the Sigma Xi has been planned in Washington, D. C., which will be known as the "D. C." Chapter. An organizing committee consisting of Marcus Benjamin (Columbia), *Chairman;* M. W. Lyon (Brown), *Secretary;* Paul Bartsch (Iowa), B. W. Everman (Indiana), Edmond Heller (Stanford), L. O. Howard (Cornell), F. J. Katz (Chicago), W. R. Maxon (Syracuse), T. S. Palmer (California), J. E. Pogue (Yale) and B. H. Ransom (Nebraska) are about to apply for a charter. As there are over 200 members in Washington it is expected that a large and flourishing chapter will be formed.

The Norman W. Harris Lectures for 1913–1914 will be delivered by Dr. Edwin Grant Conklin, professor of zoology at Princeton University, on the subject, "Heredity and Environment in the Development of Men," February 9 to 14 inclusive, Northwestern University, Evanston.

At the annual meeting of the Washington Academy of Sciences held at the Cosmos Club on January 15, the retiring president, Dr. O. H. Tittmann, delivered an address on "Our Northern Boundaries."

Professor Edward Kasner, of Columbia University, on January 17 gave a lecture at Princeton University on "Elements of Infinite Order and the Geometry of Divergent Power Series."

At an open meeting of the Sigma Xi Society at Case School of Applied Science, Cleveland, Ohio, on January 14, Dr. O. P. Hay, research associate of the Carnegie Institution of Washington, D. C., lectured on "The Ice Age of North America and its Remarkable Animals."

The first of a series of lectures on practical conservation and industrial questions, given under the auspices of the Ohio State University for the benefit of citizens of the state, was delivered January 8 by Professor C. E. Sherman, of the department of civil engineering. His theme was the regulation of streams, with special reference to floods.

On January 6 Associate Professor Frederick Starr, of the department of sociology and anthropology in the University of Chicago, begins a course of five illustrated lectures on the general subject of "Japan: The Land of the Rising Sun" at the Abraham Lincoln Center of the University Lecture Association in Chicago. The subjects of the individual lectures are as follows: "The Life of the Japanese," "Japanese Religion," "The Hairy Ainu of Japan," "Korea: The Land of the Morning Calm," and "Far Eastern Questions."

Professor Albion Woodbury Small, head of the department of sociology and anthropology in the University of Chicago, delivered on December 27, at the eighth annual meeting of the American Sociological Society in Minneapolis, his address as the retiring president of the society. The address, which was on "Problems of Social Assimilation," was given at a joint meeting of the American Sociological Society and the American Economic Association.

Mr. W. Popplewell Bloxam, formerly professor of chemistry in Presidency College, Madras, and the author of papers on the production and chemistry of indigo, died on December 26, aged fifty-three years.

Dr. George William Peckham, librarian of the Milwaukee Public Library, known for his important contributions to entomology, died on January 11, aged sixty-eight years.

Edmund B. Huey, Ph.D., died in Connell, Washington, on December 30, 1913. Dr. Huey had been in the west for a year trying to regain his health. He had previously been associated with Dr. Adolf Meyer, at the Johns Hopkins Hospital. He was the author of a book on "The Psychology of Reading" and another on "Mentally Defective Children," and was one of the foremost leaders in the more recent study of mentally defective children. He spent a year studying defective children at the State Home for the Feeble-minded at Lincoln, Ill., and had previous to this spent two years with Janet in Paris. He was preparing a book on clinical psychology, but about six months before his death the notes and what

manuscript he had prepared, the accumulation of perhaps ten years, were completely destroyed by fire.

As the result of infection by glanders bacilli while working in the laboratory, Mr. A. M. Jansen, instructor in the veterinary college of Ohio State University, died on January 4.

THE United States Geological Survey is in receipt of a cablegram from St. Petersburg in which "the Geological Survey, of Russia, announces with profound grief the unexpected death of its director, Theodosie Tchernycheff, in the fifty-seventh year of his life."

A MEMORIAL fund raised by the friends of the late Humphrey Owen Jones, F.R.S., fellow of Clare College, who, with his wife, was killed in the Alps in August, 1912, has been gratefully accepted by the university, and a Humphrey Owen Jones lectureship in physical chemistry has been established.

THE U. S. Civil Service Commission announces a competitive examination for research chemist, to fill two vacancies in this position in the Bureau of Animal Industry, Department of Agriculture, Washington, D. C., at salaries of $1,800 a year.

THE completion of the 30-inch photographic refractor of the Allegheny Observatory has been long delayed by the difficulty of manufacturing suitable glass disks. These have now been delivered by Schott and Co., of Jena, Germany, and it is expected that the telescope will be in use early next fall.

AT a recent meeting of the board of trustees of the University of Illinois, Mr. R. Y. Williams was appointed director of the miners' and mechanics' institutes, which are to be established under the direction of the department of mining engineering. Authority for the establishment of these institutes was granted by an act of the state legislature in 1911, but no appropriation was made to carry out the authorization until the latter part of the recent sesion of the legislature, at which time an appropriation of $15,000 per annum was made. The purpose of the miners' and mechanics' institutes is somewhat similar to that of the farmers' institutes, but their specific purpose is to assist men who are preparing themselves to pass the tests required by the state before they can hold official positions about the mines. Mr. Williams graduated from Princeton University in 1901.

IT is stated in the *British Medical Journal* that Dr. L. W. Sambon, who left England in August last to investigate pellagra in the West Indies, returned to London at the beginning of January. By invitation he first proceeded to the United States of America, and in Spartanburg, Columbia and Charleston he met several of the men who have recently devoted themselves to the study of pellagra, and delivered addresses before medical societies in these cities. Dr. Sambon found that in the United States the interest in the disease was very keen, owing no doubt to the evidences of the existence of pellagra to a serious extent in many parts of the country. As is well known, Dr. Sambon's opinion is that pellagra is not due to the consumption of maize, whether diseased or sound, but that it is caused by an infection brought about most probably by a fly. His investigations in southern and eastern Europe suggested that the intermediary was a simulium, an insect closely allied to the group represented by the sandfly. In the United States Dr. Sambon found many men ready to accept this hypothesis, and was greatly impressed with the work which was being carried out by the Thompson-MacFadden Pellagra Commission in South Carolina. After leaving the North American continent he proceeded to the West Indies, where he visited Jamaica, Antigua, Barbados, Trinidad, Grenada, St. Vincent and other islands. In the hospitals, asylumns and rural districts he met with many cases of pellagra, and proved the existence of the disease in several areas in which its presence had previously been unknown. Dr. Sambon also visited British Guiana, and found pellagra along the coast from the Demerara to the Berbice rivers. In part of his trip Dr. Sambon was accompanied by Captain Siler, U. S. Army, chief of the American Pellagra Commission, and by Mr. Jennings, of the Entomological Bureau, Washington, D. C.

Seventy-five per cent. of a highly valuable fertilizing material in the form of tankage and blood from the country slaughter of food animals is being wasted throughout the country districts, according to a recent bulletin of the Department of Agriculture. Tankage, a product of slaughter houses consisting of such waste material as bones, horns, hoofs, hair, etc., contains a large percentage of nitrogen and other products used in commercial fertilizer and in the larger packing houses is carefully saved. In country killing, however, only 25 per cent. of the tankage and blood are saved for fertilizer. The nitrogen content of tankage is said to vary from 5 to 8 per cent. and its phosphoric acid content between 5 and 12 per cent. Dried blood is perhaps the richest in nitrogen of all the organic materials used in the fertilizing industries. Unadulterated blood when quite dry contains 14 per cent. of nitrogen, but as obtained on the market its content varies from 9 to 13 per cent. From the figures estimated by the Bureau of Animal Industry, Department of Agriculture, as representing the total slaughter of cattle, calves, swine, and sheep in the United States, in 1912, it has been calculated that if all the materials rendered available by this slaughter had been saved and converted into tankage and dried blood, they would have produced 222,535 tons of tankage and 79,794 tons of dried blood. The introduction of a cooperative system among American farmers undoubtedly would result in an increased utilization of blood and tankage for fertilizing purposes. In Denmark country killing is being practised on a cooperative basis in small country abattoirs, and the blood is carefully preserved.

UNIVERSITY AND EDUCATIONAL NEWS

A new art building to cost $125,000 is now guaranteed for Oberlin College. The names of the donors are at their request withheld.

Mr. F. W. Bradley has offered a gift of $1,000 a year for at least ten years to endow a loan fund for students in the college of mining of the University of California. Both principal and income of the gift are to be available for these loans.

Harvard University has received the sum of $7,500 with which to establish a scholarship in memory of the late Francis Hardon Burr, '09. This fund is to be known as the Francis H. Burr 1909 Fund, and the yearly income therefrom is to be used in helping deserving undergraduates who combine as nearly as possible Burr's remarkable qualities of character, leadership and athletic ability. The fund was raised principally from the members of Burr's class, but some of his older friends also contributed.

By the will of the late Miss Emily M. Easton £10,000 are bequeathed to the Durham College of Medicine, Newcastle, and £5,000 to Armstrong College.

The dedication at the winter convocation of the University of Chicago of the new addition to the Ryerson Physical Laboratory marks a great increase in the research facilities of the university in the field of physics. The new addition is connected with the original building by corridors and consists of a basement and three floors. It contains the liquid air and refrigerating plants, the dynamos and motors, the machine and instrument shops, and the switchboard for distributing electric currents of all kinds to all parts of both buildings. It has besides two large student laboratories, a lecture room and four research rooms. The old Ryerson Laboratory has been renewed by the installation of a modern electric light and power system of unusual completeness, by the insertion of new steel-concrete floors in all the ground-floor rooms, and by the remodeling of the entire basement into a series of special research rooms, of great value where freedom from vibration and constancy of temperature are required.

The associates of Radcliffe College have elected Miss Bertha May Boody to succeed Miss Mary Coes as dean of the college. Miss Boody is a native of Brookline and received the A.B. degree from Radcliffe in 1899 and the A.M. degree from Columbia in 1912. She has

studied for one winter in the American School for Classical Studies in Rome, and for one summer in the University of Cambridge, England.

PROFESSOR WALTER MULFORD, of Cornell University, has been appointed head of the new department of forestry in the University of California. His duties will begin with August 1 next. Since there are 29,000,000 acres of national forest in California, besides vast areas of forest privately owned, the subject is one of great importance there. Dr. Patrick Beveridge Kennedy has been appointed assistant professor of agronomy. Dr. Calvin O. Esterly has been appointed as a biologist in the Scripps Institution for Biological Research at La Jolla.

MR. J. J. GALLOWAY, Ph.D. (Indiana), has been appointed instructor in geology at Indiana University.

MR. HALBERT P. BYBEE, M.A. (Indiana), has been appointed instructor in geology at the University of Texas.

MR. J. C. JOHNSON has been appointed to the chair of general biology, botany and zoology, at Auckland University College, in succession to Professor A. P. W. Thomas.

DISCUSSION AND CORRESPONDENCE

COLUMBIUM VERSUS NIOBIUM

AT a meeting of the Council of the International Association of Chemical Societies in Brussels, last September, a committee on inorganic nomenclature, among other recommendations, endorsed the name and symbol "niobium" and "Nb," for the element which was originally named columbium. As this recommendation is historically erroneous, a brief statement of the facts appears to be desirable.

In 1801, Hatchett, an English chemist, analyzed a strange American mineral, and in it found a new metallic acid; the oxide of an element which he named columbium. A year later, Ekeberg, in Sweden, analyzed a similar mineral from Finland, and discovered another element, which he called tantalum. Wollaston, in 1809, undertook a new investigation of these elements, and concluded that they were identical; a conclusion which, if it were true, would have involved the rejection of the later name, and the retention of the earlier columbium. The accepted rules of scientific nomenclature make this point clear.

For more than forty years after Hatchett's discovery, both names were in current use; for although Wollaston's views were accepted by many chemists, there were others unconvinced. In 1844, however, Heinrich Rose after an elaborate study of columbite and tantalite from many localities, announced the discovery of two new elements in them, niobium and pelopium. The latter supposed element was afterwards found to be non-existent, but the niobium was merely the old columbium under a new name. That name in some mysterious manner was substituted by the German chemists for the original, appropriate name, and has been in general use in Europe ever since. In America, the name columbium has been generally preferred, and was formally endorsed by the Chemical Section of the American Association for the Advancement of Science more than twenty years ago. In England, also, columbium is much used, as, for example, in Roscoe and Schorlemmer's "Treatise on Chemistry," Thorpe's "Dictionary of Applied Chemistry," and the new edition of the Encyclopedia Britannica.

The foundation of Rose's error seems to have been an uncritical acceptance of Wollaston's views; for he speaks of all the minerals he studied as tantalite. He also, at least in his original memoir, claims that the atomic weight of niobium is greater than that of tantalum, and here he was obviously wrong.

In short, the name columbium has more than forty years priority, and during that interval was accepted by many chemists, and was more or less in current use. To employ the name niobium is not only unhistorical, but it is also unfair to the original discoverer, meaningless, and without any justification whatever. Furthermore, it injures the splendid reputation of Rose, for it perpetuates and emphasizes one of his few errors. The recom-

mendation of the committee above mentioned should not be accepted, for it is opposed to the established rules of priority.

F. W. Clarke

THE CYTOLOGICAL TIME OF MUTATION IN TOBACCO

In the current volume of Science, p. 35, Hayes and Beinhart after describing the origin of a many-leaved variety of Cuban tobacco by mutation say:

This mutation must have taken place after fertilization, *i. e.*, after the union of the male and female reproductive cells. If the mutation had taken place in either the male or female cell before fertilization, the mutant would have been a first generation hybrid, and would have given a variable progeny the following season.

Is it not equally probable that the mutation occurred in an egg-cell which then developed without fertilization? Parthenogenesis is known to occur in tobacco, and mutation in a growing or immature germ-cell seems inherently more probable than in a fully formed and fertilized one. Perhaps the behavior of the additional mutants obtained in 1913 will throw light on the matter.

W. E. Castle

Bussey Institution,
January 2, 1913

SCIENTIFIC BOOKS

Analytical Mechanics. By Haroutune M. Dadourian, M.A., Ph.D., Instructor of Physics in the Sheffield Scientific School of Yale University. D. Van Nostrand Company. Price $3.00.

In his preface, the author states that his "work is based upon a course of lectures and recitations which the author has given, during the last few years, to the junior class of the Electrical Engineering Department of the Sheffield Scientific School." We expect this book to contain, therefore, several topics of special interest to students of electricity. We find a chapter devoted to "Fields of Force and Newtonian Potential," one to "Periodic Motion," one to "Energy" and one to "Work." But, as the author states, "In order to make the book suitable for the purposes of more than one class of students more special topics are discussed than any one class will probably take up. But these are so arranged as to permit the omission of one or more without breaking the logical continuity of the subject."

The author himself is a physicist, and perhaps he intends this book to be suitable for classes in physics. The book seems to be written from the standpoint of the physicist rather than from the standpoint of the engineer. If this book is intended for the students of civil and mechanical engineering, then it must be said it has no advantage over the number of books already in the field. I doubt if it is even as suitable.

Judging from the recent discussions concerning the teaching of mathematics and mechanics, it seems that the successful book has not yet been written. Possibly the book everybody is looking for must be written on a new plan. To say that an author deviates from the generally acknowledged plan need not be a criticism of his book. Dr. Dadourian makes his volume unique in several ways, but I doubt if it will stand the test.

In the first place, he seems to avoid the graphical treatment. The modern tendency seems to be to emphasize this phase of the subject.

The question of "units" is always a source of contention between the physicist and the engineer. The absolute system of units is certainly the most logical. To the engineer, however, it is not a question of logic, but of adaptability.

Another departure from the usual mode of procedure in modern elementary text-books in mechanics is the extent to which he makes use of "vector addition." The first chapter is devoted to the subject of the "addition and resolution of vectors." On page 10 he gives the analytical expression for the resultant of any number of vectors, and the resolution of a vector into its three rectangular components. This section is made the basis of his whole book so far as the composition and resolution of vector quantities (forces, moments, couples, etc.) are concerned. All he needs to say is,

that a couple (for instance) is a vector and the desired equations follow at once. If economy of space means "economy of thought" then the author has made his book very simple indeed. To prove the equivalence of couples it is only necessary to state: "Two couples are equal when the vectors which represent their torques are equal in magnitude and have the same direction." This follows at once from his definition of a vector.

He states in his preface "that a subject like mechanics should start with a few simple laws and the entire structure of the science should be based upon them. In the present work the following law is made the basis of the entire subject:

"*To every action there is an equal and opposite reaction, or, the sum of all the actions to which a body or a part of a body is subject at any instant vanishes.*" He further states that thus the "fundamental principle of mechanics is put in the form of a single law, which is equivalent to Newton's laws of motion and which has the advantages of the point of view involved in D'Alembert's principle."

Here is a unique departure for an elementary book. Does he mean to say that this law, whatever it may mean, is the only assumption he will make and that Newton's laws of motion as usually given will not be made use of? If he does, he completely fails. On page 16, he introduces the conception of "force" as an "action," and without any hesitation applies vector addition to a system of forces. What is he doing here, but assuming the "parallelogram of forces" in its most general form. On page 102 he assumes that a force is proportional to the accleration produced. This assumes Newton's second law of motion. In fact he makes more assumptions than are usually made in elementary text-books of mechanics.

What about the law itself? The first part of the law is clear. "To every action there is an equal and opposite reaction" is nothing but Newton's third law of motion. The word "or" leads us to think that the second part means the same thing as the first part. On page 15 he states that "The fundamental law of mechanics is known as the law of *action* and *reaction*." He then states Newton's third law and gives the following illustration. "Let us apply this law to the interaction between a book and the hand in which you hold it. Your hand presses upward upon the book in order to keep it from falling, while the book presses downward upon your hand. The law states that the action of your hand equals the reaction of the book and is in the opposite direction. The book reacts upon your hand because the earth attracts it. When your hand and the earth are the only bodies which act upon the book, the action of your hand equals and is opposite to the action of the earth. In other words, the sum of the two actions is nil. Generalizing from this simple illustration, we can put the law into the following form:

"To every action there is an equal and opposite reaction, or the sum of all the actions to which a body or a part of a body is subject at any instant vanishes."

Now does he mean to say that the pressure of the hand on the book and the force of gravity acting on the book are equal because they are action and reaction? If he does he errs. *They are not action and reaction* and *are equal only in case equilibrium exists.* He has said nothing about equilibrium and if he does not mean this then what does he mean?

On page 100 he takes up the subject of "motion of a particle." Here he says that "we must extend the meaning of the term *reaction* so as to include a form of reaction which is known as *kinetic reaction*. In his illustration we see that by *kinetic reaction* he means the so-called *force of inertia*. We also see that he considers kinetic reaction as a real force. To this, serious objections can be raised. The meaning of the second part now becomes clear. It is simply *D'Alembert's Principle*—"The impressed forces together with the reversed effective forces form a system in equilibrium."

I fail to see, however, the advantage of assuming D'Alembert's principle as a fundamental law of mechanics, especially since he finds it necessary, in reality, to assume all of Newton's laws besides. Moreover the law itself

as he states it, and his applications of it, are rather confusing. It would be difficult indeed to put D'Alembert's principle in words so that a student at the beginning of his study of mechanics could grasp its significance. Any attempt would be apt to confuse rather than help the student.

Space will not permit me to go into further detail. To be brief, he seems inclined to introduce new difficulties, and to cover up the old ones. The book is not free from loose reasoning.

E. W. Rettger
Cornell University

Conservation of Water. By Walter McCulloh, C. E. Addresses delivered in the Chester S. Lyman lecture series, 1912, before the senior class of the Sheffield Scientific School, Yale University. New Haven, Yale University Press; London, Humphrey Milford, Oxford University Press. Cloth, $6\frac{3}{4} \times 9\frac{3}{4}$ in. Pp. x + 99; 39 illustrations. $2. Postage, 15 cents.

At a time when the question of our water resources and a national policy regarding them is becoming a matter of increasing importance this book is very opportune. The lectures printed therein cover the following topics:

The first chapter considers the desirability of proper handling of our water supplies and the questions of legal jurisdiction over them. Then follows a chapter on the economic, hydrographic, topographic and geologic data necessary for an intelligent handling of the problem in any case. The third chapter gives some very interesting information in regard to the water power of the United States, both developed and undeveloped, with some statistics that are hard to collect otherwise. The value of storage reservoirs in connection with power developments is shown. The next chapter treats of water supplies for municipalities and the problems of sanitation and drainage. The last chapter describes in detail the water resources of New York state and the present important developments.

This is not a book for a specialist, already well informed in hydraulics, nor is it well adapted for use as a text-book for students. But for all readers who are interested in water resources and related problems it is a book that can be read with profit. It should be of especial value to the non-technical man who desires a broad understanding of the engineering principles involved.

R. L. Daugherty
Cornell University

Stuttering and Lisping. By E. W. Scripture. New York, The Macmillan Co. 1912. Pp. xiv + 251.

After many years of clinical work and private practise in the treatment of speech defects, Dr. Scripture has here written down many of his observations regarding the causes, symptoms and treatment of stuttering and chronic mispronunciation. As a cause of stuttering a "general anxiety neurosis" is emphasized but the author avoids much elaboration of this topic.

In the chapter on symptoms some etiology is necessarily considered and many interesting kymograph records of respiration, vocal and articulatory movements are reproduced. The method of taking these records is well illustrated by photographs, and their significance is discussed in the text. The treatment outlined is perhaps the most valuable contribution in the book and is systematically referred to the preceding diagnosis. The exercises in voice modulation—a method of treatment largely original with the author—are carefully described. Psychanalysis and suggestion are dealt with briefly.

The second part of the book treats of lisping, as it is of negligent, organic or neurotic origin. The mouth positions for articulating the different vowel and consonant sounds are indicated by diagrams, "palatograms" and photographs. Some valuable methods for inducing the patient to attain the proper positions are given. At the end of the book are fifty pages of exercises to be used in the treatment of both stuttering and lisping.

Stevenson Smith
University of Washington

SPECIAL ARTICLES

SOME PHYSIOLOGICAL OBSERVATIONS REGARDING PLUMAGE PATTERNS[1]

THIS study was undertaken with the object of carrying the analysis of the genetic factors for color pattern somewhat farther than has hitherto been done. In many forms of domestic poultry the plumage of particular parts of the body displays on each feather a definite and regular pattern. Experimental studies show that these patterns are inherited in a clean-cut Mendelian manner. In the case of the Barred Plymouth Rock color pattern, which has been more thoroughly studied in regard to its inheritance than any other single plumage pattern in birds, extensive investigations in this laboratory and elsewhere indicate that this barred pattern is represented in the gametes by a single Mendelian factor or gene. The manner in which this gene operates physiologically presents a problem of great interest, since it involves an element of morphogenetic localization.

With a view of getting further light on this matter a study has been made of the successive regeneration of feathers, in which special attention has been paid to the comparison of the pattern shown in the regenerates and in the original feather. It is the purpose here to make a preliminary statement regarding this work and some of the results obtained, to be followed later by a detailed account.

A word should be said in regard to one point of technique, since this made possible the carrying on of the investigation in a precise and critical manner. The point referred to is the method used for the identification of the individual feather follicle. If the feathers successively produced in the same follicle are to be compared, it is necessary that this particular follicle shall be capable of absolutely sure identification at any time, whether or not there is a feather present. This result was very satisfactorily attained by tattooing with india ink a circle around each follicle chosen for study. These tattoo marks are permanent throughout the life of the individual and make it possible to find at any time the follicle which one is studying.

A few of the more important results which have been obtained from this study, which has now been in progress about a year and a half, may be here set forth, as follows:

1. All feather follicles are not capable of continually producing successive feathers for an indefinite time. In the case of the general body plumage a feather is usually not regenerated more than about three times. The precise number of successive regenerations varies with different birds and different feathers. Wing primaries seem to possess the maximum regenerative capacity. After about the third removal in the case of body feathers the follicle usually remains in a perfectly quiescent condition, taking no steps whatever toward the regeneration of a new feather.

2. This failure to regenerate is, however, very definitely related to the natural moult of the bird, and in the following way. A follicle which has been absolutely inactive for a long period of time (*e. g.*, six months) preceding the natural autumn moult of the bird produces a new feather in connection with the moult, in the same manner as does any other follicle of the body. In other words the process of natural moulting reactivates the follicle which had been brought into a quiescent state by successive feather removal.

3. The precise pattern exhibited by a particular feather is, in the usual course of events, reproduced each time a feather is produced by that follicle with extreme fidelity of detail. If, however, the feather is removed from the follicle as soon as it is fully grown, thus forcing continued regenerative activity of the follicle, the pattern tends progressively to be broken up, and probably will ultimately be entirely lost as a definite pattern. The experiments have not yet gone far enough to enable us to speak positively on this latter point. A progressive breaking up of an originally definite pattern is, however, very clearly

[1] Papers from the Biological Laboratory of the Maine Agricultural Experiment Station, No. 60. This paper was read at the meeting of the American Society of Naturalists in Philadelphia, December 31, 1913.

shown in a number of cases.[2] The behavior of the color pattern in successively regenerated feathers suggests, as a working hypothesis, that the pattern factor or gene is possibly represented in each follicle by a strictly limited amount of material, and that when this is used up the pattern is lost.

4. The secondary sexual feathers of the male, such as the saddle hangers, only appear as adult plumage. The same follicles which bear these feathers produce, as juvenile plumage, undifferentiated body feathers. The formation of these secondary sexual feathers is not necessarily dependent upon any normal moult. If the juvenile feather is removed from the follicle the next feather produced by that follicle will be the secondary sexual feather, and not a feather of the juvenile type. After that all further regenerations are of the sexually differentiated feather.

These investigations are being continued. A complete report, with illustrations, covering the progress of the work to date will shortly be published elsewhere.

Raymond Pearl,
Alice M. Boring

[2] Some of which were shown in the charts used in connection with the reading of this paper.

THE AMERICAN SOCIETY FOR PHARMACOLOGY AND EXPERIMENTAL THERAPEUTICS

The fifth annual meeting of the Pharmacological Society was held in Philadelphia on Monday and Tuesday, December 29 and 30, at Jefferson Medical College and the University of Pennsylvania. The scientific meetings were auspiciously inaugurated by a joint session of the three societies which form the Federation of American Societies for Experimental Biology, comprising the Physiological Society, the Society of Biological Chemists and the Pharmacological Society. The program of this joint meeting on Monday morning was opened by a short address of the president of the Physiological Society, Dr. S. J. Meltzer, as chairman of the federation. The title of his address was "Theories of Anesthesia."

The following papers were read and discussed:

"Phlorhizin Glycosuria before and after Thyroidectomy," by Graham Lusk.

"Studies in Diabetes: (1) The Effect of Different Compounds of Glycogenesis; (2) The Mechanism of Antiketogenesis," by A. J. Ringer and E. M. Frankel (by invitation).

"Some Problems of Growth: (*a*) The Capacity to Grow; (*b*) The Rôle of Amino Acids in Growth," by L. B. Mendel and T. B. Osborne.

"Further Studies in the Comparative Biochemistry of Purine Metabolism," by Andrew Hunter.

"Changes in Fats during Absorption," by W. R. Bloor.

"Immunization against the Anti-coagulating Effect of Leech Extract," by Leo Loeb. (Read by title.)

"Anaphylaxis in the Cat and Opossum," by C. W. Edmunds.

"Vividiffusion; Report on Preliminary Results," by J. J. Abel, L. S. Rowntree and B. B. Turner.

"A Method of Dialyzing Normal Circulating Blood and Some of Its Applications," by C. L. V. Hess (by invitation) and H. McGuigan.

"A Biological Test for Iodine in the Blood," by A. Woelfel and A. L. Tatum (by invitation).

"Further Studies of the Excretion of Acids," by L. G. Henderson and W. W. Palmer (by invitation).

The second scientific session was also held at Jefferson Medical College on Monday, December 29, from 2 to 5 P.M., and the following papers were read:

"Uranium Glycosuria," by G. B. Wallace and H. B. Meyers.

"A Comparative Study of the Vascular Response of the Kidneys in Animals Nephritic from Uranium Nitrate," by W. deB. MacNider.

"The Production of Glycosuria by Zinc Salts," by W. Salant and M. Kahn.

"Further Observations on Caffein Glycosuria," by W. Salant and M. Kahn.

"Studies upon the Long-continued Feeding of Saponin," by C. L. Alsberg and C. S. Smith.

"The Effect of the Inhalation of Ether upon the Irritability of the Voluntary Peripheral Motor Mechanism," by J. Auer and S. J. Meltzer.

"The Irritability of Muscle and Motor Nerve in Chloroform Anesthesia," by T. S. Githens and S. J. Meltzer.

"The Cessation of Respiration in Deep Ether Anesthesia and its Possible Relation to the Action of Ether upon the Peripheral Motor Mechanism," by T. S. Githens and S. J. Meltzer.

"The Anesthetic Tensions of Ether Vapor for Man," by W. M. Boothby (by invitation).

"Studies in the Absorption of Drugs," by R. A. Hatcher and Cary Eggleston.

"Fatal Action of Magnesium Sulphate by Absorption from the Intestines," by J. Auer and S. J. Meltzer.

"Liberation of Formaldehyde from Hexamethylenamine in Pathological Fluids," by P. Hanzlik.

The third scientific session was held on Tuesday, December 30, from 9 to 12 M., in the engineering building of the University of Pennsylvania, and the following papers were presented and discussed:

"On Certain Effects of Digitalis Administration on the Human Heart" (with lantern demonstration), by A. E. Cohn and F. R. Fraser (by invitation).

"Quantitative Studies of Vagus Stimulation and Atropin," by J. D. Pilcher.

"Experiments on the Cardiac Action of Camphor," by O. H. Plant.

"The Action of Sodium and Potassium Iodide on the Heart and Blood Vessels," by I. D. Macht (by invitation). (Read by title.)

"The Influence of Sodium Tartrate on the Circulation," by W. Salant and C. S. Smith.

"The Pharmacological Action of Tetra-methyl-ammonium Chloride on the Circulation and Respiration," by A. S. Loevenhart.

"Two Types of Periodic Respiration Produced by Morphin," by H. G. Barbour.

"The Pharmacological Action of Certain Substances on the Lungs and Respiration," by D. E. Jackson.

"Some Further Observations on Trypan-red Iodine Compounds," by P. A. Lewis and R. B. Krauss.

"Clinical Studies with Caffein," by Lester Taylor (by invitation).

"Further Observations on the Action of Ergot," by W. Salant and S. Hecht.

"The Toxicity of Tin," by W. Salant and C. S. Smith.

On Tuesday afternoon, December 30, the three societies forming the federation held a joint meeting at the medical laboratories of the University of Pennsylvania. At this meeting only demonstrations were given and of these the demonstration of Drs. Abel, Rowntree and Turner was especially beautiful. The titles were as follows:

"The Influence of the Vagi on Renal Secretion," by R. G. Pearce.

"Stimulation of the Semi-circular Canals," by F. H. Pike.

"Demonstration of Vividiffusion," by J. J. Abel, L. G. Rowntree and B. B. Turner.

"The Determination of Blood Sugar," by P. A. Shaffer.

"Intestinal Peristalsis in Homarus," by F. R. Miller.

"Methods for Studying the Pharmacology of the Circulation," by C. Brooks.

"The Contour of the Intraventricular and the Pulmonary Arterial Pressure Curves by Two New Optically Recording Manometers," by C. J. Wiggers.

"Some Time-saving Laboratory Methods," by C. C. Guthrie.

"A Graphic Method for Recording the Coagulation of Blood," by W. B. Cannon and W. K. Mendenhall (by invitation).

"Some Mutual Relations of Oxalates, Salts of Magnesium and Calcium; Their Concurrent and Antagonistic Actions," by F. L. Gates and S. J. Meltzer.

"A Method for Obtaining Successive Contrast of the Sensations of Hunger and Appetite," by A. J. Carlson.

"Further Observations of the Pyramidal Tracts of the Raccoon and Porcupine," by S. Simpson.

"A New Apparatus for Demonstration of the Dioptrics of the Eye and the Principles of Ophthalmoscopy and Retinoscopy," by A. Woelfel.

"Simple Experiments on Respiration for the Use of Students," by Y. Henderson.

"Convenient Modification for Venous Pressure Determinations in Man," by R. D. Hooker.

"Device for Interrupting a Continuous Blast of Air, Designed Especially for Artificial Respiration," by R. A. Gesell and J. Erlanger.

"A Simple Liver Plethysmograph," by C. W. Edmunds.

"An Artificial Circulation Apparatus for Students," by W. P. Lombard.

"A Simplified and Inexpensive Oxidase Apparatus," by H. H. Bunzel.

"An Improved Form of Apparatus for Perfusion of the Excised Mammalian Heart," by M. Dressbach.

Business Meetings

Executive sessions were held by the Pharmacological Society on Monday, December 29, at 5 P.M. and on Tuesday, December 31, at 12:30 P.M. The following officers were elected for the year 1914:

President—Dr. Torald Sollmann.

Secretary—Dr. John Auer.

Treasurer—Dr. William deB. MacNider.

Additional members of the council—Dr. John J. Abel, Dr. A. L. Loevenhart.

Membership committee—Dr. Reid Hunt (term expires 1916).

Election of New Members.—The names of the following candidates were sent to the council by the membership committee, recommended for election by the council and elected by the society: Dr. A. E. Cohn, Rockefeller Institute, New York City; Dr. H. F. Helmholtz, Sprague Memorial Institute, Chicago, Ill.; Dr. W. A. Jacobs, Rockefeller Institute, New York City; Dr. Hugh MacGuigan, Northwestern Medical School, Chicago, Ill.

Federation News.—A detailed statement of the developmental effect which this first meeting of the three societies has exerted upon the federation formed at present by these societies, will be given by the general secretary of the federation for the year 1913, Dr. A. J. Carlson. Only one action, that of the Pharmacological Society, need be reported here. It will be remembered that delegates from the Physiological, Biochemical and Pharmacological societies met in Cleveland last year (1912) to establish a federation of the American societies for experimental biology. Among the motions passed unanimously was one which provided for the shifting of papers, with the author's consent, from the program of one society to that of another, if it were considered advisable by the secretaries. In order to prevent a possible conflict with the spirit of Section 2, Article III, of the constitution of the Pharmacological Society, which states that no one shall be admitted to membership who is in the permanent employ of any drug firm, a motion was put and carried unanimously by the Pharmacological Society in its executive session, recommending that no paper should be transferred to the program of the Pharmacological Society without the explicit consent of its secretary. This was done in order to prevent as far as possible the appearance of any paper of a commercial nature on the programs of the Pharmacological Society, for the other two members of the federation do not have this clause which excludes from membership those in the employ of business concerns. It may be stated that the Pharmacological Society did not take this action because of any specific occurrence, but because the society deemed it proper at this time to again emphasize its individual position in the matter.

Dinners and Smokers.—Excellent subscription dinners of very moderate cost formed an enjoyable feature of the Philadelphia meetings and were attended not only by the members of the federation, but also by the Naturalists, Zoologists and Pathologists. They were held on the evenings of December 29 and 30 at the Walton Hotel and Kugler's restaurant, respectively. There were only a few speeches; at the first dinner Drs. W. W. Keen and S. J. Meltzer spoke; at the second dinner the Naturalists presided and Dr. Raymond Pearl delivered a short address.

At the last executive session of the society a motion was passed unanimously to thank the local committee representing the University of Pennsylvania and Jefferson Medical College for the comprehensive and efficient way with which all arrangements for the meetings and the visitors' comfort were made. No names are mentioned in this expression of appreciation because the secretary is informed that practically every Philadelphia member of the three constituent societies labored on the local committee to make the first meeting of the federation as enjoyable as possible. It will be the opinion of every one present that their efforts were entirely successful, that the visitors attended with pleasure and left with regret.

John Auer,
Secretary

The Rockefeller Institute

SOCIETIES AND ACADEMIES

THE ANTHROPOLOGICAL SOCIETY OF WASHINGTON

At a special meeting of the Society held on November 4, at the National Museum, Dr. John R. Swanton read a paper entitled, "The Indian Village." He stated that while it is a common notion that country life preceded urban life, this view is not absolutely correct, urban life in its germs going back almost as far as man himself. He then took up the various factors tending to produce the village, determine its character, and subsequently knit it together. These he found to be of three orders, material, social and religious. Among the first he enumerated material available for the construction of houses, position with reference to the food supply and fresh water, and occasionally also position with reference to the sun. Among social factors he treated trade, desire for exchange of ideas, need of mutual protection and relationship, especially in the peculiar form it assumed under totemism. Finally the growth of a village or town cult was traced from the practical independence of shamanism pure and simple to the complete town ritual, sometimes directly, sometimes through the fusion of clan ceremonies

and sometimes through the rituals of religious or other societies. These factors were illustrated by reference to the tribes of the North Pacific Coast and the Gulf area. A possible evolution was suggested in three stages, first, the haphazard collection of hunters, fishers, or perhaps agriculturalists, in a certain spot; second, the development of social relations among them, particularly through intermarriage, and thirdly, a religious seal or stamp of unity, though it was not the writer's intention to set this up as a hard and fast process of evolution. It was noted that totemic clans among some tribes might have been evolved in a similar manner. In conclusion, a short comparison was made between the Indian village and the modern city, attention being called to the fact that in the latter the most important determining factor is trade, while in the former relationship, religious observances, and to some extent motives of protection, were much more prominent.

The subject was discussed at some length by Mr. J. N. B. Hewitt, who confined his remarks to the village in the social organization of the Iroquois. The basis of the social organization was actual or fictitious blood kinship traced through the mother. The cohesiveness of the several units was obtained through the ties of duty and privilege subsisting between clans united by the marriage of their sons and daughters. The clans were organized into two phratries or sisterhoods of clans, one of which represented the masculine and the other the feminine, in nature. This division was maintained in all public meetings. The one side was, therefore, called the "father side," and the other, the "child side," which of course was the "mother side." Strong lines of actual or artificial kinship and cleavage existed between these two groups.

The clans' totems have no especial religious significance at present, that is, there are no ceremonies in honor of them. That there were such in early times is quite possible. The decadence of the worship of the clan totem was probably due to the unification of the clan government into that of the tribe, and later, of the tribe into that of the confederation. The great influence of the council of women, composed of mothers only, in the affairs of the village and tribe and confederation was emphasized, and illustrated by the effectiveness with which they could stop or prevent a war. They needed only to forbid their sons to engage in warlike activity under penalty of becoming outlaws to the tribe and confederation. The gradual adoption of the Tuscarora tribe of North Carolina by the Iroquois League on motion of the Oneidas as their sponsors, was described, the Tuscaroras being first regarded as infants, then as boys who were not allowed to take part in the wars and councils of the League, and then, finally, as warriors having their own federal chiefs to represent them in the Federal Council of the League.

The 469th regular meeting of the society was held November 25, 1913, the president, Mr. Stetson, in the chair.

Dr. Daniel Folkmar, who has charge of the report on "Mother Tongue" in the Bureau of the Census, addressed the Society on "Some Results of the First Census of European Races in the United States." Statistics of the mother tongue, or native language, of the "foreign white stock" of the United States are presented in the report soon to be issued by the Bureau of the Census. It was prepared under the supervision of the chief statistician for population, assisted by the speaker as expert special agent. There are presented, for the first time in the census, figures directly relating to the ethnic composition of the white population of the United States, in so far as that is indicated by the native language. This term is taken to mean the language of customary speech in the homes of the immigrants before immigration.

One of the most interesting facts disclosed in this report is the great numerical preponderance which is still held by the mother tongues of northwestern Europe, as a whole, notwithstanding the high rank numerically which has been gained by a few individual mother tongues from eastern and southern Europe—especially the Italian, Polish and Yiddish. These three now stand third, fourth and fifth in rank. The English and Celtic mother tongues are by all odds the ones most largely represented in the foreign white stock of the United States. The number, 10,037,420, is considerably greater than that of the German mother tongue, which latter contributes more than one fourth (27.3 per cent.) of the total foreign white stock of the United States, as reported in 1910. Italian, Polish and Yiddish come next in rank, but none of them number as much as one fourth of the German. To these three mother tongues, intermediate in rank but considerable in numbers, may be added the Swedish, French and Norwegian, all belonging to northwestern Europe, except a por-

tion of the French. No other mother tongue than the eight thus far enumerated furnishes as much as 2 per cent. of the total of the foreign white stock of the United States, or numbers as much as 1,000,000. The eight major mother-tongue stocks already named account for 87.5 per cent. of the total foreign white stock.

How small a factor the "new" immigration from southern and eastern Europe really is up to the present time, may be better shown by comparing it with the total white population of the United States. Taking as 100 per cent. the total white population of the United States in 1910, numbering 81,731,957, the so-called "native stock" constitutes 60.5 per cent. and the three great linguistic families of foreign stock from northwestern Europe constitute 27.1 per cent., making a total of 87.6 per cent. The elements from southern and eastern Europe constitute, therefore, less than 13 per cent. of the total. Of this the two principal Latin mother tongues—the French and the Italian—contribute less than 5 per cent., and the two principal Slavic mother tongues—the Polish and the Bohemian—and the Hebrew, taken together, contribute also less than 5 per cent., leaving to all the remaining mother tongues another 5 per cent. or less of the total. Of the total foreign white stock of the United States, 32,243,382, there are 8,817,271 persons who are of German stock when counted according to mother tongue, but a trifle under 8,500,000 (8,495,142) of German stock when counted by their country of origin, Germany.

Immigrants from Austria are far more Slavic than Germanic. Russian immigration is shown to be far more Hebrew (52.3 per cent.) than Russian (2.5 per cent.) or even Slavic. Immigration from Turkey in Europe is not so much Turkish as Greek and Bulgarian. Both the first and the second generations of immigration from Russia show that over 50 per cent. report Yiddish and Hebrew as their mother tongue. The returns for "Yiddish and Hebrew" reflect ethnic composition less satisfactorily than the returns for other mother tongues. A part—how large a part there is no means of judging—of those whose ancestral language is Hebrew doubtless have reported German, English, Polish or other mother tongues. Of the total number of Yiddish-speaking people 838,193 came from Russia, 144,484 from Austria-Hungary, 41,342 from Roumania, 14,409 from the United Kingdom, and 7,910 from Germany.

The full list of mother tongues as reported at the Thirteenth Census is given for the total foreign white stock (which includes the foreign born and the natives of foreign or mixed parentage) and for the foreign-born whites separately, as follows:

Mother Tongue	Total Foreign Whit Stock, 1910	Foreign born White
All mother tongues	32,243,382	13,345,545
English and Celtic[1]	10,037,420	3,363,792
Germanic:		
German	8,817,271	2,759,032
Dutch and Frisian	324,930	126,045
Flemish	44,806	25,780
Scandinavian:		
Swedish	1,445,869	683,218
Norwegian	1,009,854	402,587
Danish	446,473	186,345
Latin and Greek:		
Italian	2,151,422	1,365,110
French	1,357,169	528,842
Spanish	448,198	258,131
Portuguese	141,268	72,649
Roumanian	51,124	42,277
Greek	130,379	118,379
Slavic and Lettic:		
Polish	1,707,640	943,781
Bohemian and Moravian	539,392	228,738
Slovak	284,444	166,474
Russian	95,137	57,926
Ruthenian	35,359	25,131
Slovenian	183,431	123,631
Serbo-Croatian:		
Croatian	93,036	74,036
Dalmatian	5,505	4,344
Servian	26,752	23,403
Montenegrin	3,961	3,886
Bulgarian	19,380	18,341
Slavic, n. s.	35,195	21,012
Lithuanian and Lettish	211,235	140,963
Miscellaneous:		
Yiddish and Hebrew	1,676,762	1,051,767
Magyar	320,893	229,094
Finnish	200,688	120,086
Armenian	30,021	23,938
Syrian and Arabic	46,727	32,868
Turkish	5,441	4,709
Albanian	2,366	2,312
All other	790	646
Unknown	313,044	116,272

Daniel Folkmar,
Secretary

[1] Includes persons reporting Irish, Scotch or Welsh.